Feasibility Study of Partial Observability in H_∞ Filtering for Robot localization and Mapping Problem

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***Abstract*— This paper presents H_∞ Filter SLAM, which is also known as the minimax filter to estimate the robot and landmarks location with the analysis on partial observability. Some convergence conditions are also presented to aid the analysis. Due to SLAM is a controllable but unobservable problem, it's difficult to estimate the position of robot and landmarks even though the control inputs are given to the system. As a result, Covariance Inflation which is a method of adding a pseudo positive semidefinite(PsD) matrix is proposed as one approach to analyze Partial Observability effects in SLAM and to reduce the computation cost. H_∞ Filter is capable of withstand non-gaussian noise characteristics and therefore, may provide another available approach towards SLAM solution.**

I. INTRODUCTION

A. Robotic Mapping

In achieving the task of exploration and navigation, an autonomous robot is required to collect sufficient information about the unknown environment and its surroundings conditions. One of the tasks which attempts to continuously observing landmarks and collecting information while the robot moving through an unknown environment is referred to SLAM(Simultaneous Localization and Mapping) problem, which is alternatively known as CML(Concurrent Mapping and Localization). It is believed that the SLAM problem can fully support an autonomous robot behavior. Even if the problem passed over two decades, the SLAM problem still facing a lot of unsolved tasks. SLAM becomes one of the fascinating research after some sequential series of seminal papers introduced in 1990's such as Smith and Cheeseman[1]. See Fig.1 for further explanation about SLAM.

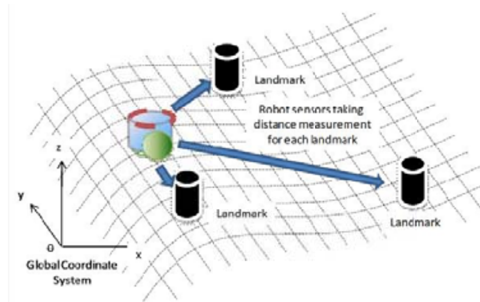

Fig. 1. Illustration for SLAM problem

SLAM has been applied in wide areas of applications, indoor or outdoor such as in satellite, mining, space exploration, rescue, and military. The development of SLAM continues whether in 2D[3] or 3D applications[4] and now expands even to home-based robot application such as the lawn moving robot and the vacuum cleaner robot. The problem is tracked historically around 1980's, which enhanced from the form of *Topological* and *Metric* approaches to *Behavioral approach*, *Mathematical-based model* approach and *Probabilistic approach*[2]. The probabilistic approach made a significant success due to its advantages, which significantly induced a level of confidence about the estimation instead according to the observations than the other two methods; mathematical models approach, which require to build a precise model, or the Behavioral approach, a method of exploiting the sensor's application to the system. In spite of remarkable achievement of probabilistic approach, there exist some shortcomings such as computational complexity. Nevertheless, with modern development of software, a considerable support and solution to this problem may exist, thus inspire further development of SLAM problem.

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Researchers around the world works to improve the SLAM performance and attempts to propose a better solution in various kinds of probabilistic techniques such as EKF, UKF, EM, Fast-SLAM. Nowadays, Fast-SLAM [2] as one of the noticeable approaches, gains a lot of attention towards SLAM development. This technique used particles as a representative of uncertainties in an unknown environment and claimed to be the best estimator as it is more robust than any other conventional approaches. If more particles are used, then the better is the estimation. However, this approach demands higher computational cost. Therefore, due to such deficiencies, such a remarkable technique does not deter the classical methods, for example, Kalman Filter and other conventional methods.

Kalman Filter, still acts as one of the famously ever applied filters in SLAM. Nonetheless, no matter what kind of filters presented above, they are still familiar and fundamentally relied on probabilistic theory. Probabilistic approaches have gained SLAM researchers great interest nowadays to model the system efficiently by considering the existence of uncertainties. The readers are encouraged to study the development of SLAM in [5], which significantly discussed the SLAM problem from various aspects.

B. Why H_∞ SLAM?

Uncertainties and dynamic environment hinder the robot to perform efficiently in most applications. These factors are the most influential terms that brought the idea of probabilistic into SLAM problem. Unfortunately, this is the most difficult problem in SLAM where all estimations are based on probabilistic. Instead of guessing with a single value, probabilistic approach introduces a set of data or information with high density, which provide a reliable data acquisition. Henceforth, the approach is applicable to most SLAM problems in most situations, especially in unknown environment with unknown noise conditions. In view to realize the truly autonomous robots, probabilistic approach is highly recommended as it allocate sufficient information to the robots for making judgement while working or operate independently in a less-human monitoring system.

Kalman Filter has been employed widely in SLAM either in linear or nonlinear SLAM case. The most fascinating factor to this statement is because Kalman Filter, which theoretically based on the MMSE approach, is easy to apply and was proved to work efficiently in most SLAM problems. Kalman Filter heavily relies on the assumption of gaussian noise, thereby suffers for a condition for non-gaussian noises. Besides, it is inappropriate to depend only for a single assumption of noise characteristics. It is a wise decision to model a system that can take into account for a worst case of noise or when the noise statistics are violated. Hence, one of the families of Kalman Filter, the H_∞ Filter can provides a better choices to tolerate with such a robust system. It assumes that the noises are bounded in certain level of energy and the designer can tune its performance to achieve a desired outcomes. As a result, the development of H_∞ Filter[6] is proposed in this paper for SLAM.

Throughout this paper, the H_∞ Filter performance in non-linear SLAM problem under two partial observable SLAM cases is studied; Unstable Partially Observable SLAM and Stable Partially Observable SLAM[7]. Partial Observability explains that, even if the robot can be controlled to move through environment, true conditions may not be the same as predicted. Unstable case describes that the estimation may go unbounded, while the stable case is able to preserve the estimation in certain level of uncertainty. H_∞ filter is still new in SLAM[8] and its convergence properties in SLAM have been shown in [9]. We carried out experiments considering a small indoor environment consisting of some point landmarks with respect to [7] and [9] to determine the convergence properties of H_∞ Filter in both partial observability cases.

This paper is organized as follows. In Section II, the general SLAM problem and H_∞ algorithm is presented with a brief comparison to Kalman Filter, while Section III explains about H_∞ convergence. Section IV discuss the decorrelation strategy based on *Covariance Inflation* method and Section V demonstrates experimental results of partially observable $H\infty$-SLAM. Finally, Section VI concludes the paper.

II. SLAM General Model

SLAM consists of two models; process model that explains how the robot move through the environment and measurement model that calculates and measures the relative distance and angle between robot and landmarks. This section analyzes both models. An assumption of stationary landmarks are made for convenience. Ther process model is presented as follows.

$$X_{k+1} = f(X_k, \omega_k, v_k, \delta\omega, \delta v) \tag{1}$$

where $X_k \in \mathbb{R}^{3+2m}, m = 1, 2, ...N$ is the augmented state consist of the robot state$\in \mathbb{R}^3$ and landmarks state $L_m \in \mathbb{R}^{2m}$. v_k, ω_k are representing the controlling terms of velocity and turning rate, and $\delta\omega$, δv are the correlated noises on v, ω respectively. On behalf of the measurement models, the following equations are presented.

$$z_i = \begin{bmatrix} r_i \\ \theta_i \end{bmatrix} = \begin{bmatrix} \sqrt{(y_i - y_{v_{k+1}})^2 + (x_i - x_{v_{k+1}})^2} + \nu_{r_i} \\ \arctan\left(\frac{y_i - y_{v_{k+1}}}{x_i - x_{v_{k+1}}}\right) - \theta_{k+1} + \nu_{\theta_i} \end{bmatrix} \tag{2}$$

where r_i, and θ_i is the relative distance and angle between robot and a landmark m. Again, the noise, $\nu_{r_i\theta_i}$ is the noise of the measurement with correlated zero mean noises of the covariance matrix $R_{r_i\theta_i}$. The prediction step is defined as

$$\hat{X}_{k+1} = f(\hat{X}_k, \omega_k, v_k, 0, 0) \tag{3}$$
$$P_{k+1} = \nabla f_X P_k \psi_k^{-1} \nabla f_X^T + \nabla f_{\omega v} \Sigma \nabla f_{\omega v}^T \tag{4}$$

$\hat{X}_k$ is the estimated augmented state. Σ_k act as the control noise $(\delta\omega, \delta v)$ covariance and f_X, $f_{\omega v}$ is shown as below.

$$\nabla f_X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -vT\sin\theta & 1 & 0 & 0 \\ vT\cos\theta & 0 & 1 & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \quad \nabla f_{\omega v} = \begin{bmatrix} \nabla f_{\omega v} \\ 0 \end{bmatrix} \tag{5}$$

and

$$\psi_k = (I_k + \nabla H_i R_k^{-1} \nabla {H_i}^T P_k - \gamma^{-2} I P_k) \tag{6}$$

∇H_i is the measurement model in Jacobian representation and shown by

$$\nabla H_i = \begin{bmatrix} 0 & -\frac{dx}{r} & -\frac{dy}{r} & \frac{dx}{r} & \frac{dy}{r} \\ -1 & \frac{dy}{r^2} & -\frac{dx}{r^2} & -\frac{dy}{r^2} & \frac{dx}{r^2} \end{bmatrix} \tag{7}$$

I is the identity matrix with an appropriate dimension. $dx = x_m - x_r$ and $dy = y_m - y_r$ and $r = \sqrt{x_m - {x_r}^2 + y_m - {y_r}^2}$. Using the Jacobian notation for a case of a robot observing one landmark, for example, an observation at point A, result in the following equation.

$$\nabla H_A = [-e \quad -A \quad A] \tag{8}$$

Denote

$$H_A = \begin{bmatrix} e & A \end{bmatrix} \qquad e = \begin{bmatrix} 0 & -1 \end{bmatrix}^T \tag{9}$$

and

$$A = \begin{bmatrix} \frac{x_m - x_A}{\sqrt{x_m - {x_A}^2 + y_m - {y_A}^2}} & \frac{y_m - y_A}{\sqrt{x_m - {x_A}^2 + y_m - {y_A}^2}} \\ \frac{y_m - y_A}{x_m - {x_A}^2 + y_m - {y_A}^2} & \frac{x_m - x_A}{x_m - {x_A}^2 + y_m - {y_A}^2} \end{bmatrix} \tag{10}$$

(x_m, y_m), (x_A, y_A) are the $m-th$ landmark coordinate and robot position at point A respectively. The initial covariance $P_0 \geq 0$ is stated as $P_0 \in \mathbb{R}^{(n+N)\times(n+N)}$ and given by

$$P_0 = \begin{bmatrix} P_{0v} & 0 \\ 0 & P_{0m} \end{bmatrix} \quad (11)$$

A. H_∞ Filter-Based SLAM

This section includes a brief introduction of H_∞ Filter-Based SLAM a comparison to Extended Kalman Filter. First, an assumption for the noise is made for each time, k.

Assumption 1: $R_k \triangleq D_k D_k^T > 0$

Assumption 2: $\sum_{t=0}^{N} \|\omega_k\|^2 < \infty, \sum_{t=0}^{N} \|v_k\|^2 < \infty$

Assumption 1 states that all measurements are correlated by noises and Assumption 2 define that both the process noise and measurement noise are bounded to a certain level of energy. The difference between Kalman Filter and H_∞ filter is shown as below. For Kalman Filter, the equation for its gain and covariance are given by

$$K_k = P_k(I + \nabla H_{ik} R_k^{-1} \nabla H_i P_k)^{-1} \quad (12)$$

$$P_{k+1} = \nabla f_{X_k} P_k (I + \nabla H_{ik}^T R_k^{-1} \nabla H_{ik} P_k)^{-1} \nabla f_{X_k}^T \quad (13)$$

On the other hand, the equation for its gain and covariance of H_∞ filter is given by

$$K_k = P_k(I - \gamma^{-2} I P_k + \nabla H_{ik}^T R_k^{-1} \nabla H_{ik} P_k)^{-1} \quad (14)$$

$$P_{k+1} = \nabla f_{X_k} P_k (I - \gamma^{-2} I P_k + \nabla H_{ik}^T R_k^{-1} \nabla H_{ik} P_k)^{-1} \times \nabla f_{X_k}^T \quad (15)$$

where process noise is assumed to be small and can be neglected. Opposite to Extended Kalman Filter(EKF), H_∞ filter(HF) depends on the covariance matrix of error signals, $Q_k \geq 0, R_k > 0$ which are designed to achieve certain desired performance. As γ values become bigger, this equation will be the same as (12),(13) of Extended Kalman Filter. In view to H_∞ Filter convergence in SLAM, [9] showed that if a robot continuously observing the same landmarks, the whole covariance matrix is converging to zero uncertainties. This seems to be unrealistic and argueable to achieve as long as noises keep interrupting the estimation. However, this is actually a result of *finite escape time*, a phenomena that occurred in H_∞ whether in the linear and nonlinear system which describes that the solution can go infinite in finite time[10]. Unlike EKF, HF solution may tend to go infinite, whether the robot is stationary or moving. Discussion about this phenomena is included in [10] therein. Some sufficient conditions for convergence are also presented in their results and are applicable to SLAM problem, which will be shown later.

Equations (13), (15) have distinctly shown the mathematical descriptions of the covariance matrix for both EKF and HF. To ensure that HF can preserve a Positive Semidefinite(PsD) matrix for all time observations, (15) must satisfy below equation.

$$\nabla H_{ik}^T R_k^{-1} \nabla H_{ik} P_k - \gamma^{-2} I P_k \geq 0 \quad (16)$$

As it can be seen, if the above γ variable is eliminated, the equation will be EKF equation that guarantees the estimations are converging for some steady state[3][12]. We then proposed below theorem about the importance of (16) in SLAM.

Theorem 1: Given that $\gamma > 0$. A stationary robot that sufficiently observed a landmark in $0 < k < \infty$, in the limit, the covariance matrix does not exhibit a *finite escape time* and bounded if and only if (16) is satisfied.

Proof: Sufficiency: Proof can be derived based on the HF algorithm about its covariance stated in (11). Given that the initial covariance is $P_0 \geq 0$. For all $k > 0$ and $\gamma > 0$, assume that $(16) \geq 0$. Then from (15),

$$P_{k+1} = \nabla f_{X_k} P_k (I - \gamma^{-2} I P_k + \nabla H_{ik}^T R_k^{-1} \nabla H_{ik} P_k)^{-1} \times \nabla f_{X_k}^T \geq 0 \quad (17)$$

As the robot is stationary, then from each update as $(17) \geq 0$,

$$P_{k+1} = [P_k^{-1} + \nabla H_{ik}^T R_k^{-1} \nabla H_{ik} P_k - \gamma^{-2} I P_k]^{-1} \quad (18)$$

$$P_{k+2} = [P_{k+1}^{-1} + \nabla H_{ik+1}^T R_{k+1}^{-1} \nabla H_{ik+1} P_{k+1} - \gamma^{-2} I P_{k+1}]^{-1} \leq P_{k+1} \quad (19)$$

Subsequently, the covariance matrix is converging to a steady-state covariance. It is also understood that, for bigger γ, above equation is approximately the same as EKF equation and finally result in the convergence of the covariance matrix.

Necessity: Consider a case where $(16) < 0$. Eventually, (17), (18) and (19) will exhibit negative covariance that is undesired properties of SLAM problem. As a result, the estimation cannot achieve the expected level of confidence. ■

We proved that if (16) is a PsD, then the covariance matrix is converging in the limit. Therefore, the convergence results from [9] are violated and the covariance matrix is guaranteed to converge to a steady state covariance. Furthermore, by (13) and (15), in the limit, the covariance matrix of HF is expected to be a slightly bigger than EKF.

Above condition is important to avoid finite escape time. However, there exists other alternative methods to avoid the problem by applying the time variant γ into the algorithm. Further explanations are included in [10]. From this point on, we demonstrates a method that able to overcome the phenomena by adopting the covariance inflation method into the filter. We believed that using this approach, the finite escape time can be avoided and at the same time realizing the reduction of cost computation for SLAM problem from $O(N^2)$ to $O(N)$. Next section discussed further about it.

III. Decorrelation using Covariance Inflation

Correlations are important[13]. To decorrelate a system, some minor changes on the covariance matrix must be done. One of the available approaches is said to be the *Covariance Inflation*. This method is used to study the effect of partial observability in two categories as follows;

- $O(N)$ but unstable partially observable H_∞-SLAM
- $O(N)$ and stable partially observable H_∞-SLAM

Decorrelation using covariance inflation is a method that adding pseudo-noise to the system. We states the mathematical description for the covariance inflation for convenience.

For H_∞ Filter, addition of pseudo noise ΔP to the HF algorithm result in

$$P_{k+1|k} = \nabla f_X P_{k|k} \psi_k^{-1} \nabla f_X^T + \Delta P_k \quad (20)$$

ψ have been shown in (7) and we assume small process noise error to the system. For 2-D realizations, based on *Covariance Inflation*, we have for $d > 0$,

$$\Delta P_k = \begin{bmatrix} dP^{12} & -P^{12} \\ -P^{21} & \frac{P^{12}}{d} \end{bmatrix} \quad (21)$$

ΔP_k is chosen to drive a smaller value of the covariance matrix, P_k. Additional details are discussed in [7]. This paper try to uncover generally the theoretical means of ΔP, which is not shown distinctly in [7][11] and can be applying both in EKF and HF. For each update, a PsD covariance matrix is given by,

$$P_{k+1} = P_k + \Delta P_k$$

where the covariance is added by a pseudo noise, ΔP_k. For the next update, it is understood that

$$\begin{aligned} P_{k+2} &= P_{k+1} + \Delta P_{k+1} \\ &= (P_k + \Delta P_k) + \Delta P_{k+1} \\ &= P_k + \Delta P_k + \Delta P_{k+1} \\ &\approx P_k + n\Delta P_k \end{aligned}$$

Lemma 1: An addition of full rank pseudo noise, ΔP to a PsD covariance matrix has no effect to the initial form of covariance update. Nevertheless, it increases the lower bound of the steady state covariance matrix by ΔP where $n = 1, ..., N$.

The above lemma show the same result obtained by *Theorem 1* in [7]. The analysis are then proceeds to understand its effect to HF algorithm. From the results of [9], the covariance matrix of a stationary robot observing one time step of one landmark at the point A is given by,

$$P_1 = \begin{bmatrix} P_{vv} & P_{vm} \\ P_{mv} & P_{mm} \end{bmatrix} \quad (22)$$

where

$$\begin{aligned} P_{vv} &= [P_{0v}^{-1} + H_A^T R_A^{-1} H_A - \gamma^{-2} I - \\ &\quad H_A^T R^{-1} A (A^T R_A^{-1} A - \gamma^{-2} I)^{-1} A^T R_A^{-1} H_A]^{-1} \\ P_{vm} &= P_{vv} H_A^T R_A^{-1} A (A^T R_A^{-1} A - \gamma^{-2} I)^{-1} \\ P_{mv} &= (A^T R_A^{-1} A - \gamma^{-2} I)^{-1} A^T R_A^{-1} H_A P_{vv} \\ P_{mm} &= (A^T R_A^{-1} A - \gamma^{-2} I)^{-1} + \\ &\quad (A^T R_A^{-1} A - \gamma^{-2} I)^{-1} A^T R_A^{-1} H_A P_{vv} \\ &\quad \times H_A^T R_A^{-1} A (A^T R_A^{-1} A - \gamma^{-2} I)^{-1} \end{aligned}$$

Each elements in (22) have been mentioned before in section II. Equation (22) exhibit one more condition in HF to be satisfied. The conditions are shown in next proposed theorem.

Theorem 2: For $\gamma > 0$, the covariance matrix of H_∞ Filter is converging to a steady state if and only if (22)≥ 0. Moreover, the following equation must be satisfied,

$$\gamma^2 I \geq R \quad (23)$$

Proof: Equation (22) shown the update of a HF and illustrate that all the elements must be a PsD for each respected update. This means that, below two equations must be satisfied.

$$\begin{aligned} P_{0v} &\geq H_A^T R^{-1} A (A^T R_A^{-1} A - \gamma^{-2} I)^{-1} A^T R_A^{-1} H_A \\ &\quad - H_A^T R_A^{-1} H_A + \gamma^{-2} I \end{aligned} \quad (24)$$

$$A^T R_A^{-1} A - \gamma^{-2} I \geq 0 \quad (25)$$

If one of these equations is not a PsD, then the covariance matrix will have a negative covariance. For a case of a stationary robot, (25) yield

$$\begin{aligned} R_A^{-1} &\geq \gamma^{-2} I \quad (26) \\ \gamma^2 I &\geq R \quad (27) \end{aligned}$$

Equation (22) also clearly stated the importance of choosing the right value of γ to obtain better estimation results for each state and (27) generally indicates that γ must be selected to be bigger than the square roots of observation noise. ■

We now understand about some conditions to ensure HF converges, which are stated in *Lemma I* and *Theorem 2* for some steady state value. The outcomes also exhibits opposite results to [9]. Besides, in linear case SLAM, (27) must be satisfied. However, it is difficult to analyze in the nonlinear system where the A matrix is changing rapidly for each different observations. Let's discuss about *Covariance Inflation.* This method adds a pseudo noise at time k, of $\Delta P_k \geq 0$ to the process model. Equation (20) has demonstrated the behavior of *Covariance Inflation* while *Theorem 1* and *Theorem 2* have sufficiently shown the results of HF convergence under some conditions. Even so, how actually the pseudo noise affects the estimations? The study now analyze its effects to HF estimation.

Theorem 3: For $\gamma > 0$ and $d > 0$, the steady-state covariance matrix, P is unboundedly increased if a full-rank pseudo noise ΔP is added recursively at each update.

Proof: In HF, if a covariance matrix is added by a full rank pseudo noise, then following equations are obtained.

$$\begin{aligned} P_k &= (P_{k-1}^{-1} + H_{k-1}^T R_{k-1}^{-1} H_{k-1} - \gamma^{-2} I)^{-1} \quad (28) \\ P_{k+1} &= P_k + \Delta P_k \geq P_k \quad (29) \end{aligned}$$

Clearly, the covariance matrix is increasing at time $k+1$ and bigger than covariance at time k. Utilizing the PsD properties of any submatrix of PsD is also a PsD, the map covariance matrix subsequently holds the same criteria as above equation and ends up to the following equation.

$$\begin{aligned} P_{k+1_{mm}} &= (P_{k+1_{mm}}^{-1} + H_k^T R_k^{-1} H_k - \gamma^{-2} I)^{-1} \quad (30) \\ &\quad + \Delta P_{k+1_{mm}} \geq P_{k+1_{mm}} \end{aligned}$$

Equations (29), (30) illustrates that the covariance inflation method may drive the covariance matrix to be unbounded and thus violating the results in [10] and Theorem 1 mentioned above. Consequently, the built map becomes more erroneous as the uncertainties increase and finally ended with unreliable estimations for both robot and landmarks location. This is the case of unstable partially observable SLAM in HF. ■
Even though *Covariance Inflation* may reduce the computation cost, the estimation may not achieve expected results. In order to prevent such a problem shown in *Theorem 3*, and concerning about [7][11], we proposed next theorem.

Theorem 4: Given $\gamma > 0$ and $d > 0$. If a pseudo-noise ΔP is added only to the landmark's covariance of H_∞ Filter at each respective update, then in the limit, the state covariance matrix is converging to

$$P^\infty \approx \begin{bmatrix} P_{vv} & P_{vm} \\ P_{mv} & P^\infty_{mm} \end{bmatrix} \quad (31)$$

where

$$P^\infty_{mm} = (A^T R_A^{-1} A - \gamma^{-2} I)^{-1} A^T R_A^{-1} H_A P_{vv} \times H^T R_A^{-1} A (A^T R_A^{-1} A - \gamma^{-2} I)^{-1} + dP_{vm} \quad (32)$$

Proof:

To ensure the boundedness of covariance matrix, robot correlation to the landmarks must be maintained while at the same time, some of the submap is set to be independent of each other. For a stationary robot observing a landmark at point A, after one-step, its covariance matrix is given by (22) with the ignorance of variable $A^{-1}R^{-1}A - \gamma^{-2}I$ for convenience(if $n \to \infty$, then $[A^{-1}R^{-1}A - \gamma^{-2}I]^{-1} \to 0$). Theoretically, when the covariance inflation is applied for one-step at the landmark's covariance, above equation is updated to the following.

$$P_1 = \begin{bmatrix} P_{vv} & P_{vm} \\ P_{mv} & P_{mm} + P_{vm} \end{bmatrix} \quad (33)$$

with $d = 1$ is set for convenience. The elements of above matrix are same to (20). Then by utilizing Matrix Inversion Lemma, the matrix inversion can be found. As the pseudo noise is added at every update with the assumption that robot has a higher initial belief than landmarks, finally the state covariance matrix consists of following matrix.

$$P^\infty_{11} = P_{vv} \quad (34)$$

$$P^\infty_{12} = P_{vm} \quad (35)$$

$$P^\infty_{21} = P_{mv} \quad (36)$$

$$P^\infty_{22} = (A^T R_A^{-1} A - \gamma^{-2} I)^{-1} A^T R_A^{-1} H_A P_{vv} \times H^T R_A^{-1} A (A^T R_A^{-1} A - \gamma^{-2} I)^{-1} + P_{vm} \quad (37)$$

Again, each elements in above matrix is shown in (22). It has been shown that from the above equations, only the map state covariance is increased to some value while the robot state covariance is maintained whenever the robot moving through the environment. ■

Above theorem proved that the robot state covariance is converged if and only if the robot state covariance is not decorrelated. These results also shows explicitly that in the limit the covariance matrix is not converging to zero as proposed in [9]. In other perspectives, it proves that the decorrelation help to avoid the *finite escape time* phenomena in HF and guarantee that covariance matrix is converging to a steady state.

IV. Experimental Results of $H_\infty SLAM$

The results of EKF-SLAM for both two cases have been studied on [7][12]. Thus, two cases of H_∞ SLAM with $\gamma = 0.9$ are studied here to understand the covariance inflation effect to H_∞ SLAM. The experiments are run in an indoor environment using an Epuck robot. Epuck robot moves through the environment while taking simultaneous observations around its surrounding. Table 1 shows the experiments control parameters.

TABLE I

Experimental Parameters

γ	0.9
Process noise, Q	$0.000001 * I_3$
Observation noise, R	0.001
Random noise observation,R	$\begin{bmatrix} R_{\theta_{max}} = 0.05 \\ R_{\theta_{min}} = -0.05 \\ R_{distance_{max}} = 0.2 \\ R_{distance_{min}} = -0.2 \end{bmatrix}$
Initial Covariance Pvv, Pmm	0.00001, 10000

A. Unstable Partially Observable H_∞ SLAM

The experimental results for this case showed consistent results to [7][11](see Figs.2-3). However, the constructed map, distinguished that the unstable H_∞ SLAM exhibits an initial stage of finite escape time. The covariance is unbounded and indicate a negative definite covariance that is unacceptable in SLAM even though its seems producing a good map. This result proves *Theorem 3*. Due to the correlation's elements and the selection of γ, the recursive update result in an impractical solutions.

As been discussed in *Theorem 2*, it is difficult to analyze the behavior of the nonlinear system as the observations are different for each landmark when the robot moves. Even though the γ can be set to have bigger value, it is afraid that in the limit, there is no difference between HF performance and KF.

B. Stable Partially Observable H_∞ SLAM

Results are shown on Figs.4-5 for the case of stable partially observable H_∞ SLAM. Although the built map has bigger uncertainties than the previous case, no finite escape time is observed in this case. This interestingly guaranteed that this case is one of the available approaches to confront the finite escape time phenomena. We perceived that HF can estimate the robot trajectory fairly. A slightly rough estimation for the landmarks can be identified. Nevertheless, it is shown that in the limit, the map covariance is converging to some steady state covariance.

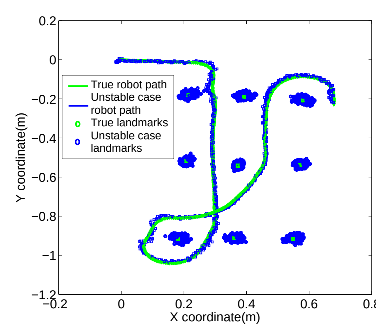

Fig. 2. Unstable partially observable H_∞ SLAM of map construction

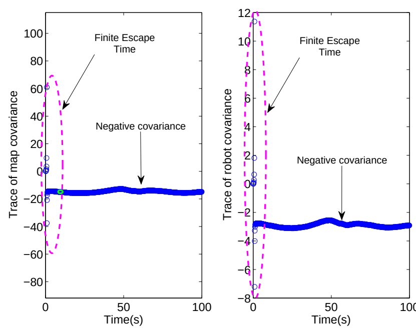

Fig. 3. Robot covariance while observing landmarks of unstable partially observable SLAM

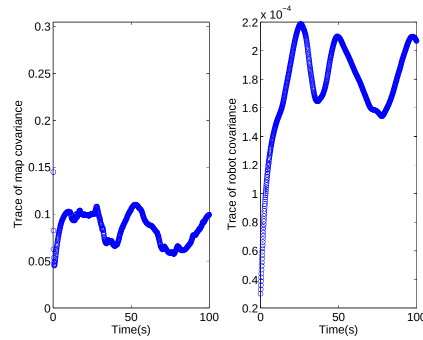

Fig. 5. Robot covariance while observing landmarks of stable partially observable SLAM

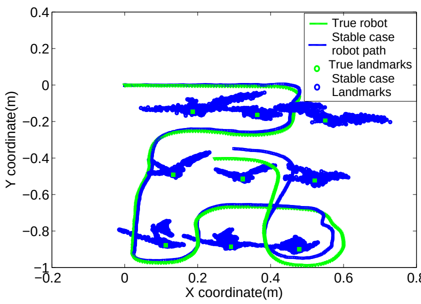

Fig. 4. Stable partially observable H_∞ SLAM of map construction

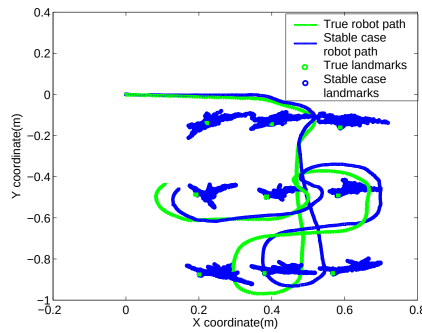

Fig. 6. Stable Partially Observable SLAM under uniform noise distribution

V. Conclusion

We have shown that the decorrelation algorithm may reduce the computation cost and may result in unbounded convergence as shown by the unstable partially observable SLAM problem. On the other hand, preserving the robot correlations with the landmarks while some of the submaps are decorrelate from others, may decrease the uncertainties and ensure the convergence and may avoid the finite-escape time phenomena if some conditions are fulfilled.

A. Future Works

Further analysis of HF condition in the nonlinear system to achieve a steady state covariance is one of the important things to make HF as another available solution for SLAM problem.

B. Acknowledgments

The authors gratefully acknowledged Kementerian Pengajian Tinggi Malaysia(KPT) and University Malaysia Pahang(UMP) for their continuous support and comments.

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