A Sensor Network Configuration Considering Priori Estimation Error and Communication Energy

Takashi Takeda * **and Toru Namerikawa** **

* *Division of Electrical Engineering and Computer Science, Graduate School of Natural Science and Technology, Kanazawa University, Kakuma-machi Kanazawa, 920-1192, JAPAN (e-mail: ttakashi@scl.ec.t.kanazawa-u.ac.jp).*
** *Department of System Design Engineering, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, JAPAN (e-mail: namerikawa@sd.keio.ac.jp)*

Abstract: This paper deals with a network configuration considering priori estimation error variance and communication energy in sensor networked feedback system. Each sensor transmits information to a fusion center via multi-hop communication network. We propose a novel network configuration algorithm that each sensor node transmits information with a same dimension. The transmitted information is merged in each sensor node and consists of measurement data with time delay. Then the priori estimate can be minimum variance estimation. The proposed algorithm achieves sub-optimal network topology with minimum energy and a desired variance. Experimental results show an effectiveness of the proposed method.

Keywords: sensor network, communication energy, data fusion, priori estimation error variance, multi-hop communication

1. INTRODUCTION

Recently, much attention has been attracted to a wireless sensor network. It generally consists many sensor nodes connected wirelessly with memory units, communications and calculation capabilities, Shi et al. (2008). It is well known that sensor networks are superior in a fault tolerance, sensing in broad area, collection and application of information etc. there are applications to a environmental monitoring, security and intelligent system. Additionally, its application to not only a sensing system but also configuration of the feedback control system via a sensor network and large scale online information processing has received attention in the areas of traffic control, nano-medicines and disaster countermeasures, Shi et al. (2008).

Meanwhile, each sensor node requires electric power more than a case of only sensing because of communications and calculations, but sensor nodes are generally powered and driven by batteries. Moreover it is difficult to change batteries frequently or charge by power cable because of increasing of costs. Therefore, it is important to utilize the energy efficiently to achieve the energy-saving and prolong sensor nodes life in Arai et al. (2009). For this requirement, the sensor scheduling, the optimization of the communication rate and communications traffic and decreasing communication distances by the multi-hop communication are discussed by Iino et al. (2008), Arai et al. (2009), Shi et al. (2007). Consequently, in this paper, we discuss a multi-hop network configuration problem considering the estimation error variance and communication energy in a feedback control system via a sensor network.

The estimation problem in a sensor network system has been studied in Olfati-Saber et al. (2008), Nebot et al. (1999), Song et al. (2007), Sandberg et al. (2008). A distributed Kalman filtering algorithm with a consensus strategy were proposed in Olfati-Saber (2005, 2007), Carli et al. (2007). In these methods each sensor node communicates with its neighbors on a network. However, if the plant is applied control inputs from fusion center or one of sensor nodes, all sensor node have to obtain its information in real time and it is difficult to develop real system.

In Shi et al. (2007), a network configuration problem with a multi-hop communication and a feedback control system considering communication energy and estimation error variance has been considered. However amount of information transmitted from each sensor node increase with a number of sensor nodes.

In this paper, we discuss a network configuration problem considering the priori estimation error variance and communication energy in a feedback control system via a sensor network. We first define a sensor network with multi-hop communication. Then we assume that each sensor node transmit same amount of information for issue resolution of increasing amount of information transmitted. In this system, we discuss a estimation problem and a network configuration problem. Then we show that there is the unique positive definite solution to the discrete algebraic Riccati equation in the error covariance update

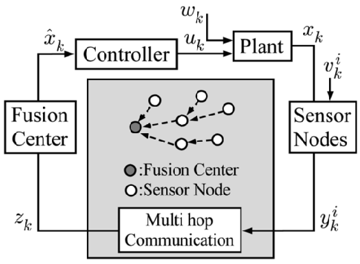


Fig. 1. Sensor network system.

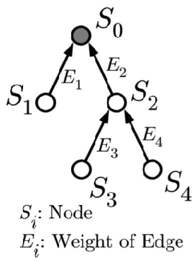


Fig. 2. An example of network.

and a trade-off between the estimation error variance and a communication energy. Secondly, we propose a network configuration algorithm considering this trade-off. This network configuration algorithm achieves sub-optimal network topology with minimum energy and a desired error variance. Finally, we verify effectiveness of a sensor scheduling algorithm by experiments.

This paper is organized as follows. The feedback control system via a sensor network and the network topology are presented and problems are formulated accordingly in Section II. Section III describes a novel information fusion algorithm, a estimation algorithm and the unique solution to the discrete algebraic Riccati equation under some assumptions. A network configuration algorithm is proposed in Section IV. Finally, some experimental results are presented in Section V.

2. PROBLEM FORMULATION

2.1 Plant and Sensor Nodes

In this paper, we consider the feedback control system via a sensor network illustrated in Fig. 1. This system consists the plant and N sensor nodes S_i, $(i = 1, 2, ..., N)$. We assume all sensor nodes can take a measurement of the plant. The process dynamics of the plant and the measurement equation of a sensor node S_i are given by

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{1}$$

$$y_k^i = C_i x_k + v_k^i \tag{2}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$, $y_k^i \in \mathbb{R}^{q_i}$ are the state, the control input and the measurement output of a sensor node S_i respectively. Additionally, $w_k \in \mathbb{R}^n$, $v_k^i \in \mathbb{R}^{q_i}$ are the process noise and measurement noise respectively. From (2), each sensor node takes a different measurement. Moreover, (1) and (2) satisfy the following *assumptions 1-3.*

Assumption 1. w_k and $v_k = \left[(v_k^1)^{\mathrm{T}} \ (v_k^2)^{\mathrm{T}} \ \cdots \ (v_k^N)^{\mathrm{T}} \right]^{\mathrm{T}} \in \mathbb{R}^q$, $(q = \sum_{i=1}^N q_i)$ are zero mean white Gaussian noise and satisfy the following equations

$$\mathrm{E} \left\{ \begin{bmatrix} w_k \\ v_k \end{bmatrix} \begin{bmatrix} w_k^{\mathrm{T}} & v_k^{\mathrm{T}} \end{bmatrix} \right\} = \begin{bmatrix} Q & \mathbf{0} \\ \mathbf{0} & R \end{bmatrix}, \tag{3}$$

$$\mathrm{E} \left\{ w_k x_0^{\mathrm{T}} \right\} = \mathbf{0}, \mathrm{E} \left\{ v_k x_0^{\mathrm{T}} \right\} = \mathbf{0}, \tag{4}$$

where Q is a positive semidifinite matrix and $R = \mathrm{diag}(R_1, R_2, ...)$ is a positive definite matrix.

Assumption 2. $(A, Q^{\frac{1}{2}})$ is reachable.

Assumption 3. (C, A) is detectable, where

$$C = \left[C_1^{\mathrm{T}} \ C_2^{\mathrm{T}} \ \cdots \ C_N^{\mathrm{T}} \right]^{\mathrm{T}}. \tag{5}$$

Assumptions 2 and 3 are required for a positive definite unique solution of a Riccati equation defined later.

2.2 Network Topology

In this paper, we deal this problem as a multi-hop communication. N sensor nodes and the fusion center S_0 are connected wirelessly and information transmitted from each sensor node are passed on to the fusion center via some relay nodes. The example of a network topology is illustrated in Fig. 2. In this paper, let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ denoted a graph with the set of vertices $\mathcal{V}$ and the set of edges $\mathcal{E}$. Then sensor node S_i and network topology satisfy the following *Assumptions 4, 5.*

Assumption 4. Sensor nodes S_i can transmit $z_k^i \in \mathbb{R}^r$ to the other sensor node once per time step with the constant communication delay less than a sampling time. Additionally, when a sensor node S_i transmit information, this sensor node uses the energy E_i.

Assumption 5. A network topology T is a directed spanning tree with root S_0.

From *Assumption 5*, a sensor node S_i transmits z_k^i containing information of a measurement of S_i to the other sensor node. The dimension of z_k^i is r in all sensor nodes. Moreover, each sensor node use a energy E_i for transmitting z_k^i to other sensor node [1]. We assume the following E as a total energy for the whole system.

$$E = \sum_{i=1}^{N} E_i. \tag{6}$$

Remark 1. Information entropy or information energy of a multi-agent system has been studied in Parunak et al. (2001), Tatikonda et al. (1999). In Parunak et al. (2001), information energy depend on the probability that each agent measure the system state. But in our framework, measurement output is avairable at all time steps. Thus, this paper do not deals with information energy or information entropy.

The energy E_i is the weight of the edge of the network topology T. In general, the communication energy depend

[1] The communication energy E_i generally can be as $E_i = b_i + a_i(d_i)^{c_i}$, where b_i is a static part and a_i is a dynamic part. c_i is typically from 2 through 6, Shi et al. (2007). The communication energy depend on a distance of a communication pass.

on a length of a communication pass between sensor nodes S_i and S_0. Consequently, if there are some relay node between S_i and S_0, the communication energy to pass to the sensor node S_0 from S_i will be reduced. But all sensor nodes transmit information once per one time step and the time delay between sensor nodes S_i and S_0 will increase. Consequently, there is a trade-off between an estimation accuracy and a communication energy.

2.3 Control Problems

In this paper, we discuss an estimation problem and a network configuration problem of sensor network feedback system.

Problems can be formulated as following *Problems 1, 2*.

Problem 1. We assume the plant and all sensor nodes satisfy *Assumptions 1-5* and the network topology T is given. Then compute the optimal state estimate $\hat{x}_k^-$ that minimizes the following estimation error variance.

$$J = \mathrm{E}\left\{\left(x_k - \hat{x}_k^-\right)^{\mathrm{T}}\left(x_k - \hat{x}_k^-\right)\right\} \tag{7}$$

Problem 2. Find the optimal network topology T^* satisfying $J \leq \gamma$, *Assumption 5* and the following equation:

$$T^* = \arg\min_{T} E, \tag{8}$$

where $\gamma > 0$ is a design parameter.

3. PROPOSED METHOD

3.1 Information merge method

In this paper, we define the sensor node receiving information from a sensor node S_i as the sensor node $\mathrm{Par}(S_i)$ and the set including sensor nodes transmitting information to a sensor nodeS_i as the set $\mathcal{N}_i = \{j|\mathrm{Par}(S_j) = S_i\}$. Moreover we define the depth h_i of a sensor node S_i, the hight $\bar{h} = \max_i h_i$ of the network topology T. For example, $\mathcal{N}_0 = \{1, 2\}$ and $\bar{h} = 2$ in Fig. 2.

A measurement output of each sensor node S_i have to merge via z_k^i with same dimension. Consequently, we propose following information fusion method for each sensor node.

$$z_k^i = C_i^{\mathrm{T}} R_i^{-1} y_{k-\bar{h}+h_i}^i + \sum_{j\in\mathcal{N}_i} z_{k-1}^j, \tag{9}$$

where $y_k^i = y_0^i, (k \leq 0)$. A dimension of $C_i^{\mathrm{T}} R_i^{-1} y_{k+h_i-\bar{h}}^i$ is n and all sensor nodes transmit information with same dimension. Moreover we propose a following information fusion method for fusion center.

$$z_k = \sum_{i\in\mathcal{N}_0} z_k^i, \tag{10}$$

where $z_k^0 = z_k$ is in the fusion center which merges all information. It follows from $y_{k-\bar{h}+h_i}^i$ and z_{k-1}^j, $(j \in \mathcal{N}_i)$ in (9) that z_k^i delays 1 time step per one relay node. Consequently, in a network topology with *Assumption 5*, information z_k merged in the fusion center is given by the following equation.

$$\begin{aligned} z_k &= \sum_{j\in\mathcal{N}_0} z_k^j \\ &= \sum_{j=1}^{N} C_j^{\mathrm{T}} R_j^{-1} y_{k-\bar{h}+1}^j \end{aligned} \tag{11}$$

z_k is calculated in the fusion center at time step k and includes $C_i R_i^{-1} y_{k-\bar{h}+1}^i$ of all sensor nodes. The time step of measurements belonging to z_k depends on $\bar{h}$. The bigger $\bar{h}$ is, the bigger a time delay of measurement belonging to z_k.

3.2 State Estimation Algorithm

We showed fusion center calculate z_k including measurements with delay $y_{k-\bar{h}+1}^i$ at time step k. In this section, we propose a estimation algorithm using z_k. Then a estimation algorithm satisfies following *Theorem 1* in a sensor network system (1) and (2).

Theorem 1. Consider the system (1), (2) and network topology T with *Assumption 1-5*. Then a estimation algorithm is given by following equations and the estimate $\hat{x}_k^j$ is minimum variance estimate based measurements of sensor node S_j:

$$\hat{x}_k^- = A^{\bar{h}-1}\hat{x}_{k-\bar{h}+1} + \bar{B}_{\bar{h}}\bar{u}_{k-\bar{h}+1}, \tag{12}$$

$$\begin{aligned}\hat{x}_{k-\bar{h}+1} &= \hat{x}_{k-\bar{h}+1}^- \\ &\quad + P_{k-\bar{h}+1}\left(z_k - C^{\mathrm{T}}R^{-1}C\hat{x}_{k-\bar{h}+1}^-\right),\end{aligned} \tag{13}$$

$$P_k^- = A^{\bar{h}-1}P_{k-\bar{h}+1}\left(A^{\bar{h}-1}\right)^{\mathrm{T}} + G_{\bar{h}}\bar{Q}G_{\bar{h}}^{\mathrm{T}}, \tag{14}$$

$$P_{k-\bar{h}+1} = \left\{\left(P_{k-\bar{h}+1}^-\right)^{-1} + C^{\mathrm{T}}R^{-1}C\right\}^{-1}, \tag{15}$$

where $\bar{B}_{\bar{h}}$, $\bar{G}_{\bar{h}}$ and $\bar{Q} \in \mathbb{R}^{n(\bar{h}-1)\times n(\bar{h}-1)}$ are as follows

$$\bar{B}_{\bar{h}} = \left[\, B \;\; AB \;\; \cdots \;\; A^{\bar{h}-2}B \,\right], \tag{16}$$

$$G_{\bar{h}} = \left[\, I_n \;\; A \;\; \cdots \;\; A^{\bar{h}-2} \,\right], \tag{17}$$

$$\bar{Q} = \text{block diag}\{Q, Q, ..., Q\}, \tag{18}$$

$$\bar{u}_{k-\bar{h}+1} = \left[\, u_{k-1}^{\mathrm{T}} \;\; u_{k-2}^{\mathrm{T}} \;\; \cdots \;\; u_{k-\bar{h}+1}^{\mathrm{T}} \,\right]^{\mathrm{T}}. \tag{19}$$

Proof. First define the following fictitious measurement output $y_{k-\bar{h}+1}$,

$$\begin{aligned} y_{k-\bar{h}+1} &= \left[\, (y_{k-\bar{h}+1}^1)^{\mathrm{T}} \;\; (y_{k-\bar{h}+1}^2)^{\mathrm{T}} \;\; \cdots \;\; (y_{k-\bar{h}+1}^N)^{\mathrm{T}} \,\right]^{\mathrm{T}} \\ &= Cx_{k-\bar{h}+1} + v_{k-\bar{h}+1}. \end{aligned} \tag{20}$$

$y_{k-\bar{h}+1}$ includes measurements taken at time step $k - \bar{h} + 1$ of all sensor nodes. Then we consider the estimation algorithm using $y_{k-\bar{h}+1}$ taken at time step k. The equation (1) can be rewritten as follows

$$x_k = A^{\bar{h}-1}x_{k-\bar{h}+1} + \bar{B}_{\bar{h}}\bar{u}_{k-\bar{h}+1} + G_{\bar{h}}\bar{w}_{k-\bar{h}+1}, \tag{21}$$

where $\bar{u}_{k-\bar{h}+1}$, $\bar{w}_{k-\bar{h}+1}$ are as follows

$$\bar{u}_{k-\bar{h}+1} = \begin{bmatrix} u_{k-1} \\ u_{k-2} \\ \vdots \\ u_{k-\bar{h}+1} \end{bmatrix}, \; \bar{w}_{k-\bar{h}+1} = \begin{bmatrix} w_{k-1} \\ w_{k-2} \\ \vdots \\ w_{k-\bar{h}+1} \end{bmatrix}. \tag{22}$$

(21) is difference equation of time step k and $k-\bar{h}+1$. Then we propose the following estimation algorithm for (21) and (20).

$$\hat{x}_k^- = A^{\bar{h}-1}\hat{x}_{k-\bar{h}+1} + \bar{B}_{\bar{h}}\bar{u}_{k-\bar{h}+1} \tag{23}$$

$$\hat{x}_{k-\bar{h}+1} = \hat{x}_{k-\bar{h}+1}^- + K_{k-\bar{h}+1}\Big(y_{k-\bar{h}+1} - C\hat{x}_{k-\bar{h}+1}^-\Big) \tag{24}$$

where $\hat{x}_k^- = \mathrm{E}\{x_k|y_0, y_1, ..., y_{k-\bar{h}+1}\}$ and $\hat{x}_{k-\bar{h}+1} = \mathrm{E}\{x_{k-\bar{h}+1}|y_0, y_1, ..., y_{k-\bar{h}+1}\}$ are estimations of x_k and $x_{k-\bar{h}+1}$ based all measurements up to time step $k-\bar{h}+1$. Now, the estimation error variance J is given by the following equation.

$$J = \mathrm{E}\left\{\left(x_k - \hat{x}_k^-\right)^{\mathrm{T}}\left(x_k - \hat{x}_k^-\right)\right\} = \mathrm{tr}P_k^- \tag{25}$$

It follows from (21), (23) and (24) that the error covariance matrix P_k and the filter gain K_k satisfying $\frac{\partial}{\partial K_k}\mathrm{tr}P_k^- = \mathbf{0}$ are as follows

$$K_{k-\bar{h}+1} = P_{k-\bar{h}+1}C^{\mathrm{T}}R^{-1} \tag{26}$$

$$P_{k-\bar{h}+1} = \left\{\left(P_{k-\bar{h}+1}^-\right)^{-1} + C^{\mathrm{T}}R^{-1}C\right\}^{-1} \tag{27}$$

Meanwhile, error covariance matrix P_k^- is as follow

$$P_k^- = A^{\bar{h}-1}\left\{\left(P_{k-\bar{h}+1}^-\right)^{-1} + C^{\mathrm{T}}R^{-1}C\right\}^{-1}\left(A^{\bar{h}-1}\right)^{\mathrm{T}} + G_{\bar{h}}\bar{Q}G_{\bar{h}}^{\mathrm{T}}, \tag{28}$$

where $\bar{Q}$ is covariance matrix of $\bar{w}_{k-\bar{h}+1}$. Consequently, a estimation algorithm using a measurement output (20).

Secondly, we show this algorithm is a estimation algorithm using z_k in (9).

It follows from (26), (20) and (24) that we can get following.

$$\hat{x}_{k-\bar{h}+1} = \hat{x}_{k-\bar{h}+1}^- + P_{k-\bar{h}+1}\Big(z_k - C^{\mathrm{T}}R^{-1}C\hat{x}_{k-\bar{h}+1}\Big). \tag{29}$$

(29) is a estimation algorithm using z_k merged in the fusion center. These equations complete the proof.

3.3 Relation between an estimation error variance and a network topology

In this section, we consider a relation between the estimation error variance $\mathrm{tr}P_k^-$ and the network topology. It follows from *Assumptions 2, 3* that there is the unique positive definite solution $P_\infty^{\bar{h}}$ to algebraic Riccati equation (14) satisfying the following equation:

$$P_\infty^{\bar{h}} = A^{\bar{h}-1}\left\{\left(P_\infty^{\bar{h}}\right)^{-1} + C^{\mathrm{T}}R^{-1}C\right\}^{-1}\left(A^{\bar{h}-1}\right)^{\mathrm{T}} + G_{\bar{h}}\bar{Q}G_{\bar{h}}^{\mathrm{T}}. \tag{30}$$

From (30), the solution $P_\infty^{\bar{h}}$ depend on the depth $\bar{h}$. Now the solution $P_\infty^{\bar{h}}$ satisfies following *Theorem 2*.

Theorem 2. We assume if $\bar{h} = \alpha, \beta$, $(\alpha > \beta)$, there is the unique positive definite solutions P_∞^α, P_∞^β to algebraic Riccati equation (14) respectively. Then P_∞^α and P_∞^β satisfy following relation:

$$\mathrm{tr}P_\infty^\alpha \geq \mathrm{tr}P_\infty^\beta. \tag{31}$$

Proof. It follows from *Assumptions 2* and *3* that the solution to (30) do not depend on initial value. Moreover (14) is different equation between k and $k-\bar{h}-1$. Consequently it is apparent from these.

From *Theorem 2*, The smaller $\bar{h}$ is, the smaller priori estimation error is. Consequently, there is trade-off between an estimation error variance and communication energy. In next section, we propose a network configuration algorithm based *Theorem 2*.

4. NETWORK CONFIGURATION ALGORITHM

In this section, we discuss a network configuration algorithm. We have to configurate a network topology satisfying $J = \mathrm{tr}P_\infty^- \leq \gamma$ and *Assumption 5*. For this purpose, we first need to calculate $\bar{h}$ satisfying $J = \mathrm{tr}P_\infty^{\bar{h}} \leq \gamma$. secondly, we find rooted spanning tree where depths of all sensor node are less than $\bar{h}$ and a communication energy E is minimized. This tree is known as $\bar{h}$-HMST(the minimum-cost $\bar{h}$-hop spanning tree). In several researches, they showed approximation algorithm and it is difficult to solve this problem optimally, Althaus et al. (2005).

In this paper, we propose an algorithm minimizing in a subset of all available network topology. Minimizing in all available network topology means that we get the optimal solution of $\bar{h}$-HMST problem or *Problem 2*. Minimizing in a subset of all available network topology means the suboptimal solution of *Problem 2*. We first consider following operation.

- Change destination of sensor nodes receiving information from sensor nodes belonging the set $\mathcal{V}_1$ into sensor nodes belonging the set $\mathcal{V}_2$,

where $\mathcal{V}_1 = \left\{S_j | h_j > \bar{h}\right\}$ and $\mathcal{V}_2 = \left\{S_j | h_j < \bar{h}\right\}$. It follows from this operation that all sensor nodes have depths with less than $\bar{h}$. We assume the set of all available network topology that we can get from this operation as $\mathcal{T}_s$. We rewrite *Problem 2* to following problem.

Problem 3. Find the optimal network topology T^* satisfying $J \leq \gamma$, *Assumption 5* and following equation:

$$T = \underset{T \in \mathcal{T}_s}{\arg\min}\, E, \tag{32}$$

where $\gamma > 0$ is a design parameter.

In *Problem 2* we find the network topology minimizing a communication energy in all available network topology. However *Problem 3* minimize in the subset of all available network topology.

We propose *Network Configuration Algorithm* and it is a solution of *Problem 3*.

Network Configuration Algorithm

1: Compute of h satisfying the following

$$J = \mathrm{tr} P_\infty^{\bar{h}} \le \gamma.$$

2: Compute rooted minimum spanning tree T by Prim's algorithm and define

$$\mathcal{V}_1 = \left\{S_j | h_j > \bar{h}\right\}, \mathcal{V}_2 = \left\{S_j | h_j < \bar{h}\right\}.$$

3: Change $\mathrm{Par}(S_i)$, $(S_i \in \mathcal{V}_1)$

if $\mathcal{V}_1$ is not an empty set

$$\mathrm{Par}(S_i) := \arg\min_{S_j \in \mathcal{V}_2} e\left(S_i, S_j\right)$$
$$E_i := e\left(S_i, S_j\right)$$
$$h_i := h_j + 1$$

end

4: **return** T

In this algorithm, we use Prim's Algorithm finding the minimum spanning tree. In *network configuration algorithm*, $e(S_i, S_j)$ is communication energy between sensor nodes S_i and S_j. *Network Construction Algorithm* satisfying following theorem.

Theorem 3. Network Construction Algorithm minimize a communication energy E in subset $\mathcal{T}_s$ and it is the solution of *Problem 3.*

Proof. In **3:** of *Network Construction Algorithm*, we select a sensor node with minimum communication energy belonging to the set $\mathcal{V}_2$. Because the operation are applied these sensor nodes, this algorithm is the solution of *Problem 3.*

Consequently, by designing γ, we can configurate a network topology what are superior to estimation accuracy or communication energy.

5. EXPERIMENTAL EVALUATION

In this section, an effectiveness of a sensor scheduling algorithm is evaluated by experiments. The experiment was carried out on a two-wheeled vehicle, a CCD camera and a computer as shown in Fig. 3. Now the two-wheeled vehicle has the nonholonomic constraint. However the two-wheeled vehicle can be defined following framework by virtual structure for feedback linearization, Namerikawa et al. (2008).

$$A = \begin{bmatrix} 1 & 0 & \delta & 0 \\ 0 & 1 & 0 & \delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} \delta^2/2 & 0 \\ 0 & \delta^2/2 \\ \delta & 0 \\ 0 & \delta \end{bmatrix}, \tag{33}$$

where $\delta = 0.2$ and $x_0 = [\,1.3\ 0.7\ 0\ 0\,]^{\mathrm{T}}$ are the sampling time and the initial state respectively. Additionally, we design the feedback gain L by LQG control. There are ten sensor nodes available and each sensor nodes has the following measurement equation and these position is shown in Fig. 4.

$$y_k^i = [\,1\ 0\ 0\ 0\,]\, x_k + v_k^i, \quad (i = 1, 2)$$
$$y_k^i = [\,0\ 1\ 0\ 0\,]\, x_k + v_k^i, \quad (i = 3, 4)$$
$$y_k^i = [\,0\ 0\ 1\ 0\,]\, x_k + v_k^i, \quad (i = 5, 6)$$

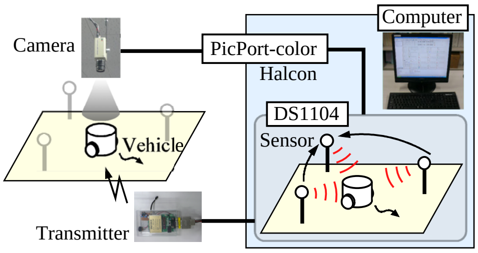


Fig. 3. Experimental setup

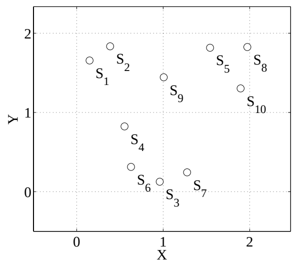


Fig. 4. Position of sensor nodes

$$y_k^i = [\,0\ 0\ 0\ 1\,]\, x_k + v_k^i, \quad (i = 7, 8)$$
$$y_k^i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x_k + v_k^i, \quad (i = 9, 10)$$

Each measurement output is calculated from the image of a CCD camera mounted above the vehicle. The video signals are acquired by a frame grabber board PicPort-color and image processing software HALCON generate nine measurements. Consequently, nine sensor nodes, a network topology and measurement noises exist in the computer. We use DS1104 (dSPACE Inc.) as a real-time calculating for an estimation and sensor scheduling. Additionally, the covariance matrices of noises are $Q = 1 \times 10^{-4} I_4$, $R = 0.05 I_{12}$ respectively.

Here we define the communication energy between arbitrarily two sensor nodes. We assume that the communication energy between sensor nodes S_i and S_j is $e_{i,j} = \epsilon d_{i,j}^2$. d_{i,f_k} is the distance between sensor nodes S_i and S_j and ϵ is the positive constant.

Additionally, experiments were done following *Case 1* and *Case 2.*

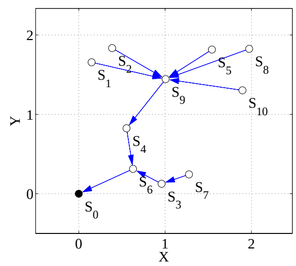
Case 1 : The experiment designing $\gamma = 0.015$
Case 2 : The experiment designing $\gamma = 0.03$

The experimental results of *Case 1* and *Case 2* are shown in Fig. 5, 6. Fig. 5, 6(a), (b), (c) and (d) show a network topology, the state x_k, the estimate $\hat{x}_k$ and a information variable z_k respectively. As shown in Figs. 5(a), 6(a), network topologies satisfying the condition are $\bar{h} = 4, 6$ respectively. Additionally, error variances are J =0.0297, 0.0137 and communication energy are $E = 5.24\epsilon, 3.12\epsilon$ respectively. Consequently, there is a trade-off between an estimation accuracy and a communication energy. As shown in Figs. 5(c), 6(c), a vibration of the estimate in *case 1* is smaller than *Case 2.* Fig. 7 shows the variance $J = \mathrm{tr} P_k^-$ in *Case 1* and *Case 2* respectively. As shown in Fig. 7, $\mathrm{tr} P_k^-$ converge on $\mathrm{tr} P_k^-$ =0.0297, 0.0137 and it is less than the design parameters respectively.

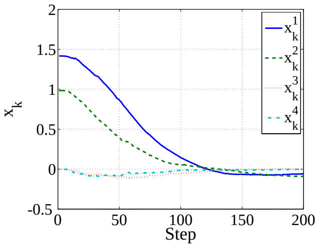
Consequently, we have showed that we can configurate a network topology what are superior to estimation accuracy or communication energy by designing γ.

6. CONCOLUTION

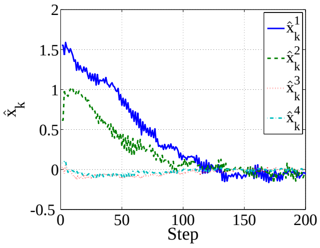
In this paper, we discussed a network configuration problem considering the priori estimation error variance and communication energy in a feedback control system via a sensor network. We first have defined a sensor network with multi-hop communication. Then we have assumed that each sensor node transmit same amount of information for issue resolution of increasing amount of information transmitted. Then we showed that there is the unique positive definite solution to the discrete algebraic Riccati equation in the error covariance update and a trade-off between the estimation error variance and a communication energy. Secondly, we have proposed a network configuration algorithm considering this trade-off. This network configuration algorithm achieves sub-optimal network topology with minimum energy and a desired error variance. Finally, we have verified effectiveness of a sensor scheduling algorithm by experiments.



(a) Network topology (Case 1).

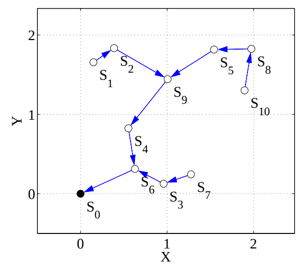


(b) State x_k (Case 1).

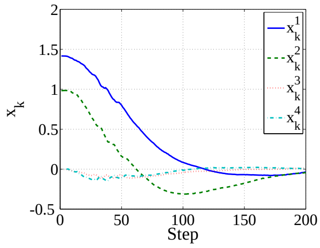


(c) Estimate $\hat{x}_k$ (Case 1).

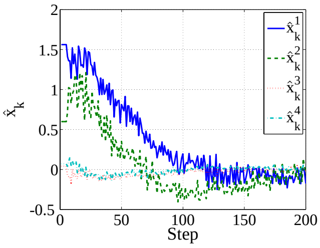
Fig. 5. Experimental results (Case 1).



(a) Network topology (Case 2).



(b) State x_k (Case 2).



(c) Estimate $\hat{x}_k$ (Case 2).

Fig. 6. Experimental results (Case 2).

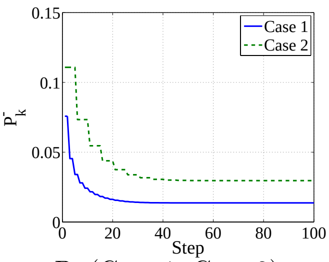


Fig. 7. Variance trP_k^- (Case 1, Case 2).

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