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Bilateral Teleoperation of Wheeled Mobile Robot with Time Delay using Virtual Image Robot

Yasunori Kawai, Toru Namerikawa and Masayuki Fujita

***Abstract*— This paper considers the bilateral teleoperation of wheeled mobile robot with time delay using the virtual image robot. In this paper, we introduce the virtual image robot as a master robot. The human operator commands the virtual image robot, the slave wheeled mobile robot tracks the motion of the virtual image robot. The kinematics and dynamics of master/slave robots is considered. The scattering theory and passivity based control schemes for bilateral teleoperation is applied, the control law is proposed. In the simulation, the performance of the proposed bilateral teleoperation system is verified using wheeled mobile robots.**

I. INTRODUCTION

Teleoperation consists of a dual robot system called a master robot and a slave robot. For the command of the human operator, the slave robot situated at a remote location tracks the motion of the master robot. In order to improve the task performance, the force feedback from the slave robot to the master robot is needed. In this way, the teleoperation is said to be controlled bilaterally [1].

The master and slave robots are connected to the network in the bilateral teleoperation. The time delay can be imposed in transmission of the data between the master and the slave site. It is known that the time delay in the closed loop system destabilize the stable system. In previous researches, scattering theory and passivity-based control are used to guarantee the stability in case the time delay exists [2], [3], [4], [5].

In previous researches, the teleoperation of a wheeled mobile robot is considered [6]. The human operator control the slave mobile robot by operating a master haptic joystick. The passivity based control of bilateral teleoperation [7] is applied, the stability is guaranteed. Instead of using the scattering theory, Lyapunov-Krasovskii functional for delayed system is proposed.

In this paper, the bilateral teleoperation of the slave wheeled mobile robot with time delay is considered. The proposed bilateral teleoperation system has the virtual image robot as the master robot. It is convenient for the human operator to look the virtual image robot compared to using only the haptic joystick. It needs that the stability is guaranteed between the virtual image robot and the wheeled mobile robot situated at a remote location. This paper utilizes the scattering theory and passivity based control schemes of the teleoperation, where the scattering transformation proposed in previous research [2] is applied. However, the differences between [2] and this paper are shown as follows. The dimension of the state vector is multidimensional. This paper considers not only the dynamics but also kinematics. This paper proposes the control law. It is shown that the stability of the bilateral teleoperation systems is guaranteed by using Lyapunov function. Though the kinematics includes nonlinear equations, design parameters of the proposed controller are obtained from the constant matrix inequality. In the simulation, the performance of the proposed bilateral teleoperation system is verified using wheeled mobile robots.

The organization of this paper is as follows. The modeling and problem formulation is shown Section 2. In Section 3, the stability of the bilateral teleoperation is presented. In Section 4, simulation results are indicated. Finally, our conclusions are presented.

Y. Kawai is with the Department of Electrical Engineering, Ishikawa National College of Technology, Ta-1 Kitachujo, Tsubata, Ishikawa 920-0392 JAPAN y_kawai@ishikawa-nct.ac.jp

T. Namerikawa is with the Department of System Design Engineering Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522 JAPAN namerikawa@sd.keio.ac.jp

M. Fuita is with Department of Mechanical and Control Engineering, Tokyo Institute of Technology, Tokyo, 152-8550 JAPAN fujita@ctrl.titech.ac.jp

II. MODELING AND PROBLEM FORMULATION

Consider the wheeled mobile robot shown in Fig. 1. The coordinate of the wheeled mobile robot consists of the position (x_m, y_m) and the rotation θ_m. The control inputs are given by the velocity v_m and steering angle $\dot{\theta}_m$. The kinematics model of the mobile robot is derived as follows

$$\frac{d}{dt}\begin{bmatrix} x_m \\ y_m \\ \theta_m \end{bmatrix} = \begin{bmatrix} \cos\theta_m & 0 \\ \sin\theta_m & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_m \\ \dot{\theta}_m \end{bmatrix}. \tag{1}$$

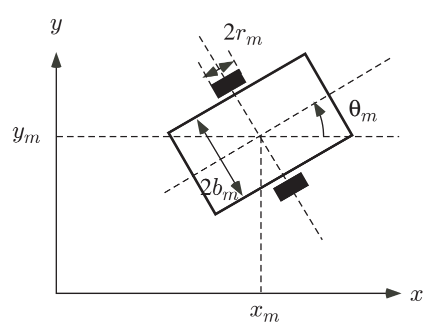

Fig. 1. Wheeled Mobile Robot

The velocity v_m and steering angle $\dot{\theta}_m$ are related to the angle velocities of the right and left wheels ω_{m1}, ω_{m2}

$$\begin{bmatrix} v_m \\ \dot{\theta}_m \end{bmatrix} = \begin{bmatrix} \frac{r_m}{2} & \frac{r_m}{2} \\ \frac{r_m}{2b_m} & -\frac{r_m}{2b_m} \end{bmatrix} \begin{bmatrix} \omega_{m1} \\ \omega_{m2} \end{bmatrix} \tag{2}$$

where r_m, b_m is the radius of the wheels and the half-width of the wheeled mobile robot, respectively.

From the equations (1), (2), the kinematics model of the mobile robot is derived as follows

$$\frac{d}{dt}\begin{bmatrix} x_m \\ y_m \\ \theta_m \end{bmatrix} = \begin{bmatrix} \frac{r_m}{2}\cos\theta_m & \frac{r_m}{2}\cos\theta_m \\ \frac{r_m}{2}\sin\theta_m & \frac{r_m}{2}\sin\theta_m \\ \frac{r_m}{2b_m} & -\frac{r_m}{2b_m} \end{bmatrix} \begin{bmatrix} \omega_{m1} \\ \omega_{m2} \end{bmatrix}. \quad (3)$$

The dynamics of the mobile robot is written as

$$\begin{bmatrix} J_{m1} & 0 \\ 0 & J_{m2} \end{bmatrix} \begin{bmatrix} \dot{\omega}_{m1} \\ \dot{\omega}_{m2} \end{bmatrix} + \begin{bmatrix} c_{m1} & 0 \\ 0 & c_{m2} \end{bmatrix} \begin{bmatrix} \omega_{m1} \\ \omega_{m2} \end{bmatrix} = \begin{bmatrix} \tau_{m1} \\ \tau_{m2} \end{bmatrix} \quad (4)$$

where J_{m1}, J_{m2} are the right and left moment of inertia, c_{m1}, c_{m2} are the right and left viscous friction, τ_{m1}, τ_{m2} is the right and left torque, respectively.

For the above equations (3), (4), we consider the teleoperation of the slave wheeled mobile robot by using the virtual image robot shown in Fig. 2. The operator gives the torque of the virtual image robot by the joystick, the force from environment is received through the joystick. As if the operator take a ride in the virtual image robot, the operator control the slave wheeled mobile robot. The control object is that the slave wheeled mobile robot moves as the same as the virtual image robot.

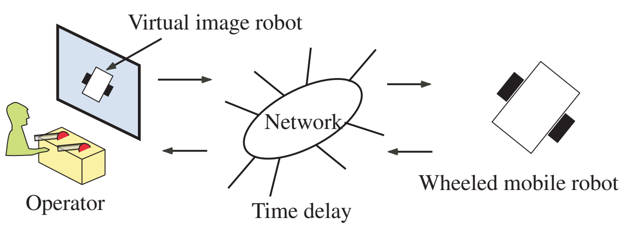

Fig. 2. Bilateral Teleoperation using Virtual Image Robot

III. STABILITY OF BILATERAL TELEOPERATION

The bilateral teleoperation system is shown in Fig. 3. The bilateral teleoperation system is composed of the following parts: the human operator, the master, the communication block, the slave, the environment and the control parts. The human operator commands the master with force F_h to move it with angle velocity ω_m. The position and the rotation of the robot information q_m is sent to the slave side. The local controller F_{feed} on the slave side drives the control input that the position q_s and the angle velocity ω_s equal to the position q_m and the velocity ω_m of the master. If the slave contacts a remote environment, the force F_e affects the position and the velocity of the slave. The local controller F_{back} on the master side gives the control input which decrease the error of the position and the angle velocity between the master and the slave.

The master virtual image model is given as follows

$$\dot{q}_m = S_m(q_m)\omega_m \quad (5)$$

$$J_m\dot{\omega}_m + c_m\omega_m = \tau_m \quad (6)$$

$$\tau_m = F_h + F_{back} - F_m \quad (7)$$

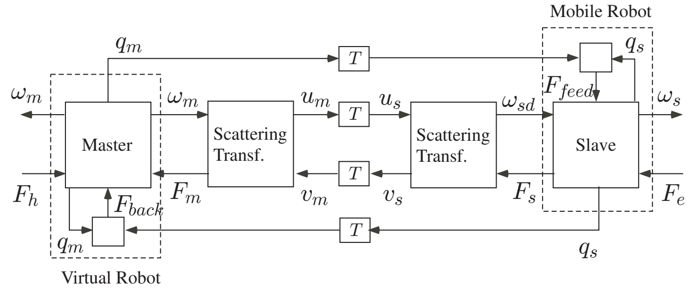

Fig. 3. Bilateral Teleoperation System

where q_m, ω_m, $S_m(q_m)$, J_m, c_m, τ_m are defined

$$q_m = \begin{bmatrix} x_m \\ y_m \\ \theta_m \end{bmatrix}, \quad \omega_m = \begin{bmatrix} \omega_{m1} \\ \omega_{m2} \end{bmatrix}$$

$$S_m(q_m) = \begin{bmatrix} \frac{r_m}{2}\cos\theta_m & \frac{r_m}{2}\cos\theta_m \\ \frac{r_m}{2}\sin\theta_m & \frac{r_m}{2}\sin\theta_m \\ \frac{r_m}{2b_m} & -\frac{r_m}{2b_m} \end{bmatrix}$$

$$J_m = \begin{bmatrix} J_{m1} & 0 \\ 0 & J_{m2} \end{bmatrix}, \quad c_m = \begin{bmatrix} c_{m1} & 0 \\ 0 & c_{m2} \end{bmatrix}, \quad \tau_m = \begin{bmatrix} \tau_{m1} \\ \tau_{m2} \end{bmatrix}.$$

In a similar way, the slave model is written as

$$\dot{q}_s = S_s(q_s)\omega_s \quad (8)$$

$$J_s\dot{\omega}_s + c_s\omega_s = \tau_s \quad (9)$$

$$\tau_s = F_s + F_{feed} - F_e \quad (10)$$

where q_s, ω_s, $S_s(q_s)$, J_s, c_s, τ_s are given

$$q_s = \begin{bmatrix} x_s \\ y_s \\ \theta_s \end{bmatrix}, \quad \omega_s = \begin{bmatrix} \omega_{s1} \\ \omega_{s2} \end{bmatrix}$$

$$S_s(q_s) = \begin{bmatrix} \frac{r_s}{2}\cos\theta_s & \frac{r_s}{2}\cos\theta_s \\ \frac{r_s}{2}\sin\theta_s & \frac{r_s}{2}\sin\theta_s \\ \frac{r_s}{2b_s} & -\frac{r_s}{2b_s} \end{bmatrix}$$

$$J_s = \begin{bmatrix} J_{s1} & 0 \\ 0 & J_{s2} \end{bmatrix}, \quad c_s = \begin{bmatrix} c_{s1} & 0 \\ 0 & c_{s2} \end{bmatrix}, \quad \tau_s = \begin{bmatrix} \tau_{s1} \\ \tau_{s2} \end{bmatrix}.$$

The scattering transformation is described as

$$u_m = \frac{1}{\sqrt{2b}}(F_m + b\omega_m), \; v_m = \frac{1}{\sqrt{2b}}(F_m - b\omega_m)$$

$$u_s = \frac{1}{\sqrt{2b}}(F_s + b\omega_{sd}), \; v_s = \frac{1}{\sqrt{2b}}(F_s - b\omega_{sd}) \quad (11)$$

The slave side receives the information of the master T [s] ago. The signal $u_s(t)$ is equal to the signal $u_m(t-T)$. Similarly, $v_m(t) = v_s(t-T)$.

The some assumptions are introduced, which is assumed in [2].

- The human operator and the environment can be modeled as passive systems.
- The operator and the environmental force are bounded by known functions of the master and the slave velocities respectively.
- All signals belong to the L_{2e}, the extended L_2 space.
- The angle velocities ω_m and ω_s equal zero for $t < 0$.

The local control law is proposed.

$$F_{back} = S_m^T K(q_s(t-T) - q_m) \quad (12)$$

$$F_{feed} = S_s^T K(q_m(t-T) - q_s) \quad (13)$$

$$F_s = B_{s2}(\omega_{sd} - \omega_s) \quad (14)$$

$$K = \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_1 & 0 \\ 0 & 0 & K_2 \end{bmatrix} \quad (15)$$

where $K_1 > 0,\ K_2 > 0,\ K_3 > 0$. The following theorem is proposed.

The position tracking error between the master and the slave robot is defined as

$$e_s = q_m(t-T) - q_s(t) \quad (16)$$

where $q_m(t-T)$ is the delayed master position and rotation received on the slave side.

Theorem 1: Consider the system described by (5)-(10) and the control law (12)-(15). The design parameter K is obtained from the following inequality

$$T^2(S_s^T K S_s)(S_m^T K S_m) < c_s c_m. \quad (17)$$

The master and slave angle velocities asymptotically converge to the origin, the position error (16) remains bounded.

Proof: Define a positive definite function for the system as

$$\begin{aligned} V \;=\; & \frac{1}{2}\omega_m^T J_m \omega_m + \frac{1}{2}\omega_s^T J_s \omega_s + \frac{1}{2}(q_m - q_s)^T K (q_m - q_s) \\ & + \int_0^t \left(\omega_s^T F_e - \omega_m^T F_h\right) d\tau + \int_0^t \left(\omega_m^T F_m - \omega_{sd}^T F_s\right) d\tau. \end{aligned} \quad (18)$$

The human operator and remote environment are passive

$$\int_0^t \omega_s^T F_e d\tau \geq 0,\ -\int_0^t \omega_m^T F_h d\tau \geq 0. \quad (19)$$

Using the scattering transformation (11), the communication block is passive

$$\int_0^t \left(\omega_m^T F_m - \omega_{sd}^T F_s\right) d\tau = \frac{1}{2}\int_{t-T}^t \left(u_m^2 + v_m^2\right) d\tau \geq 0. \quad (20)$$

Thus, the function V is positive-definite. The derivative of (18) along the trajectories of the system is given by

$$\begin{aligned} \dot{V} \;=\; & \omega_m^T J_m \dot{\omega}_m + \omega_s^T J_s \dot{\omega}_s + (\dot{q}_m - \dot{q}_s)^T K (q_m - q_s) \\ & + \omega_s^T F_e - \omega_m^T F_h + \omega_m^T F_m - \omega_{sd}^T F_s \\ =\; & \omega_m^T(-c_m\omega_m + \tau_m) + \omega_s^T(-c_s\omega_s + \tau_s) \\ & + (S_m\omega_m - S_s\omega_s)^T K (q_m - q_s) \\ & + \omega_s^T F_e - \omega_m^T F_h + \omega_m^T F_m - \omega_{sd}^T F_s \\ =\; & \omega_m^T\left[-c_m\omega_m + F_h + S_m^T K(q_s(t-T) - q_m) - F_m\right] \\ & + \omega_s^T\left[-c_s\omega_s + F_s + S_s^T K(q_m(t-T) - q_s) - F_e\right] \\ & + (\omega_m^T S_m^T - \omega_s^T S_s^T) K (q_m - q_s) \\ & + \omega_s^T F_e - \omega_m^T F_h + \omega_m^T F_m - \omega_{sd}^T F_s \\ =\; & -\omega_m^T c_m \omega_m + \omega_m^T F_h + \omega_m^T S_m^T K q_s(t-T) \\ & -\omega_m^T S_m^T K q_m - \omega_m^T F_m \\ & -\omega_s^T c_s \omega_s + \omega_s^T F_s + \omega_s^T S_s^T K q_m(t-T) \\ & -\omega_s^T S_s^T K q_s - \omega_s^T F_e \\ & +\omega_m^T S_m^T K q_m - \omega_m^T S_m^T K q_s - \omega_s^T S_s^T K q_m + \omega_s^T S_s^T K q_s \\ & +\omega_s^T F_e - \omega_m^T F_h + \omega_m^T F_m - \omega_{sd}^T F_s \\ =\; & -\omega_m^T c_m \omega_m - \omega_s^T c_s \omega_s + \omega_s^T F_s - \omega_{sd}^T F_s \\ & +\omega_m^T S_m^T K(q_s(t-T) - q_s) + \omega_s^T S_s^T K(q_m(t-T) - q_m) \\ & -\omega_m^T S_m^T K q_m + \omega_m^T S_m^T K q_m - \omega_s^T S_s^T K q_s + \omega_s^T S_s^T K q_s \\ =\; & -\omega_m^T c_m \omega_m - \omega_s^T c_s \omega_s + \omega_s^T F_s - (\omega_{sd} - \omega_s + \omega_s)^T F_s \\ & +\omega_m^T S_m^T K(q_s(t-T) - q_s) + \omega_s^T S_s^T K(q_m(t-T) - q_m) \\ =\; & -\omega_m^T c_m \omega_m - \omega_s^T c_s \omega_s - (\omega_{sd} - \omega_s)^T F_s \\ & +\omega_m^T S_m^T K(q_s(t-T) - q_s) + \omega_s^T S_s^T K(q_m(t-T) - q_m) \\ =\; & -\omega_m^T c_m \omega_m - \omega_s^T c_s \omega_s - (\omega_{sd} - \omega_s)^T B_{s2}(\omega_{sd} - \omega_s) \\ & +\dot{q}_m^T K(q_s(t-T) - q_s) + \dot{q}_s^T K(q_m(t-T) - q_m) \end{aligned} \quad (21)$$

Using the following relation

$$q_i(t-T) - q_i = -\int_0^T \dot{q}_i(t-\tau)d\tau, \quad i = m, s \quad (22)$$

the equation (21) is transformed

$$\begin{aligned} \dot{V} \;=\; & -\omega_m^T c_m \omega_m - \omega_s^T c_s \omega_s - (\omega_{sd} - \omega_s)^T B_{s2}(\omega_{sd} - \omega_s) \\ & -(K^{\frac{1}{2}}\dot{q}_m)^T \int_0^T K^{\frac{1}{2}}\dot{q}_s(t-\tau)d\tau \\ & -(K^{\frac{1}{2}}\dot{q}_s)^T \int_0^T K^{\frac{1}{2}}\dot{q}_m(t-\tau)d\tau. \end{aligned} \quad (23)$$

Integrating the above the equation on the time interval $[0, t_f]$

$$\begin{aligned} \int_0^{t_f} \dot{V} dt \;=\; & -c_m\|\omega_m\|_2^2 - c_s\|\omega_s\|_2^2 - B_{s2}\|\omega_{sd} - \omega_s\|_2^2 \\ & -\int_0^{t_f}(K^{\frac{1}{2}}\dot{q}_m)^T \int_0^T K^{\frac{1}{2}}\dot{q}_s(t-\tau)d\tau dt \\ & -\int_0^{t_f}(K^{\frac{1}{2}}\dot{q}_s)^T \int_0^T K^{\frac{1}{2}}\dot{q}_m(t-\tau)d\tau dt \end{aligned} \quad (24)$$

where the notation $\|\cdot\|_2$ represents the L_2 norm of a signal on the interval $[0,\ t_f]$. Using Schwartz inequality for any $\alpha_1 > 0,\ \alpha_2 > 0$

$$\begin{aligned} & 2\int_0^{t_f}(K^{\frac{1}{2}}\dot{q}_m)^T \int_0^T K^{\frac{1}{2}}\dot{q}_s(t-\tau)d\tau dt \\ & \leq \alpha_1 \int_0^{t_f} \dot{q}_m^T K \dot{q}_m dt + \frac{1}{\alpha_1}\int_0^{t_f}\int_0^T \dot{q}_s(t-\tau)^T K \dot{q}_s(t-\tau)d\tau dt \\ & \leq \alpha_1 \left\|K^{\frac{1}{2}}\dot{q}_m\right\|_2^2 + \frac{T^2}{\alpha_1}\left\|K^{\frac{1}{2}}\dot{q}_s\right\|_2^2. \end{aligned} \quad (25)$$

Similarly,

$$\begin{aligned} & 2\int_0^{t_f}(K^{\frac{1}{2}}\dot{q}_s)^T \int_0^T K^{\frac{1}{2}}\dot{q}_m(t-\tau)d\tau dt \\ & \leq \alpha_2 \left\|K^{\frac{1}{2}}\dot{q}_s\right\|_2^2 + \frac{T^2}{\alpha_2}\left\|K^{\frac{1}{2}}\dot{q}_m\right\|_2^2. \end{aligned} \quad (26)$$

The equation (24) becomes as follows

$$\begin{aligned}\int_0^{t_f} \dot{V} dt \quad &\leq \quad -c_m\|\omega_m\|_2^2 - c_s\|\omega_s\|_2^2 - B_{s2}\|\omega_{sd}-\omega_s\|_2^2 \\ &\quad +\frac{\alpha_1}{2}\left\|K^{\frac{1}{2}}\dot{q}_m\right\|_2^2 + \frac{T^2}{2\alpha_1}\left\|K^{\frac{1}{2}}\dot{q}_s\right\|_2^2 \\ &\quad +\frac{\alpha_2}{2}\left\|K^{\frac{1}{2}}\dot{q}_s\right\|_2^2 + \frac{T^2}{2\alpha_2}\left\|K^{\frac{1}{2}}\dot{q}_m\right\|_2^2 \\ &\leq \quad -c_m\|\omega_m\|_2^2 - c_s\|\omega_s\|_2^2 - B_{s2}\|\omega_{sd}-\omega_s\|_2^2 \\ &\quad +\frac{\alpha_1}{2}\left\|K^{\frac{1}{2}}S_m\omega_m\right\|_2^2 + \frac{T^2}{2\alpha_1}\left\|K^{\frac{1}{2}}S_s\omega_s\right\|_2^2 \\ &\quad +\frac{\alpha_2}{2}\left\|K^{\frac{1}{2}}S_s\omega_s\right\|_2^2 + \frac{T^2}{2\alpha_2}\left\|K^{\frac{1}{2}}S_m\omega_m\right\|_2^2 \\ &\leq \quad -c_m\|\omega_m\|_2^2 - c_s\|\omega_s\|_2^2 - B_{s2}\|\omega_{sd}-\omega_s\|_2^2 \\ &\quad +\left(\frac{\alpha_1}{2}S_m^TKS_m + \frac{T^2}{2\alpha_2}S_m^TKS_m\right)\|\omega_m\|_2^2 \\ &\quad +\left(\frac{\alpha_2}{2}S_s^TKS_s + \frac{T^2}{2\alpha_1}S_s^TKS_s\right)\|\omega_s\|_2^2. \end{aligned} \quad (27)$$

If the following inequalities are satisfied

$$\left(\frac{\alpha_1}{2}+\frac{T^2}{2\alpha_2}\right)S_m^TKS_m < c_m \quad (28)$$

$$\left(\frac{\alpha_2}{2}+\frac{T^2}{2\alpha_1}\right)S_s^TKS_s < c_s \quad (29)$$

$\dot{V} < 0$ is derived. It can be shown that $S_m^TKS_m$ is constant matrix independent of θ_m.

$$\begin{aligned} S_m^TKS_m &= \begin{bmatrix} \frac{r_m}{2}\cos\theta_m & \frac{r_m}{2}\sin\theta_m & \frac{r_m}{2b_m} \\ \frac{r_m}{2}\cos\theta_m & \frac{r_m}{2}\sin\theta_m & -\frac{r_m}{2b_m} \end{bmatrix} \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_1 & 0 \\ 0 & 0 & K_2 \end{bmatrix} \\ &\quad \times \begin{bmatrix} \frac{r_m}{2}\cos\theta_m & \frac{r_m}{2}\cos\theta_m \\ \frac{r_m}{2}\sin\theta_m & \frac{r_m}{2}\sin\theta_m \\ \frac{r_m}{2b_m} & -\frac{r_m}{2b_m} \end{bmatrix} \\ &= \begin{bmatrix} K_1\left(\frac{r_m}{2}\right)^2(\cos^2\theta_m+\sin^2\theta_m)+K_2\left(\frac{r_m}{2b_m}\right)^2 & K_1\left(\frac{r_m}{2}\right)^2(\cos^2\theta_m+\sin^2\theta_m)-K_2\left(\frac{r_m}{2b_m}\right)^2 \\ K_1\left(\frac{r_m}{2}\right)^2(\cos^2\theta_m+\sin^2\theta_m)-K_2\left(\frac{r_m}{2b_m}\right)^2 & K_1\left(\frac{r_m}{2}\right)^2(\cos^2\theta_m+\sin^2\theta_m)+K_2\left(\frac{r_m}{2b_m}\right)^2 \end{bmatrix} \\ &= \begin{bmatrix} K_1\left(\frac{r_m}{2}\right)^2+K_2\left(\frac{r_m}{2b_m}\right)^2 & K_1\left(\frac{r_m}{2}\right)^2-K_2\left(\frac{r_m}{2b_m}\right)^2 \\ K_1\left(\frac{r_m}{2}\right)^2-K_2\left(\frac{r_m}{2b_m}\right)^2 & K_1\left(\frac{r_m}{2}\right)^2+K_2\left(\frac{r_m}{2b_m}\right)^2 \end{bmatrix} \end{aligned} \quad (30)$$

Similarly, $S_s^TKS_s$ is also constant matrix.

We can solve the condition that the inequalities (28), (29) have positive solutions α_1, α_2. From the inequality (28)

$$\alpha_1 I < 2c_m(S_m^TKS_m)^{-1} - \frac{T^2}{\alpha_2}I \quad (31)$$

and the inequality (29) becomes

$$\begin{aligned} \frac{1}{\alpha_1}I \quad &< \quad \frac{2}{T^2}c_s(S_s^TKS_s)^{-1} - \frac{\alpha_2}{T^2}I \\ \alpha_1 I \quad &> \quad \left(\frac{2}{T^2}c_s(S_s^TKS_s)^{-1} - \frac{\alpha_2}{T^2}I\right)^{-1} \end{aligned} \quad (32)$$

where symbol I means a unit matrix. Using the results (31), (32)

$$\left(\frac{2}{T^2}c_s(S_s^TKS_s)^{-1} - \frac{\alpha_2}{T^2}I\right)^{-1} < 2c_m(S_m^TKS_m)^{-1} - \frac{T^2}{\alpha_2}I \quad (33)$$

is satisfied. The following result is derived

$$\alpha_2^2 I - 2\alpha_2 c_s(S_s^TKS_s)^{-1} + T^2(S_m^TKS_m)c_m^{-1}c_s(S_s^TKS_s)^{-1} > 0. \quad (34)$$

Using the relation $(c_s(S_s^TKS_s)^{-1})^T = c_s(S_s^TKS_s)^{-1}$, the following matrix inequality is obtained.

$$T^2(S_s^TKS_s)(S_m^TKS_m) < c_s c_m. \quad (35)$$

Thus, if the matrix K satisfies the inequality (35), then matrix inequalities (28), (29) also are satisfied. From the derivative of the Lyapunov function $\dot{V} < 0$, the signal ω_m, ω_s, $q_m - q_s$ are bounded. Using the same way as [2], the master/slave acceleration and $\dot{\omega}_{sd}$ are bounded. The asymptotic convergence of the angle velocities ω_m, ω_s are guaranteed. The position tracking error (16) can be rewritten as

$$e_s = q_m(t) - q_s(t) - \int_{t-T}^{t} \dot{q}_m(\tau)d\tau. \quad (36)$$

Thus, the position tracking error is bounded. ■

IV. SIMULATION

In this section, the performance of the bilateral teleoperation proposed in section 3 is verified. The model parameters are given as $J_{m1} = J_{m2} = 1.5$ [kgm^2], $C_{m1} = C_{m2} = 0.5$ [Nms], $r_m = 0.1$ [m], $b_m = 0.3$ [m], $J_{s1} = J_{s2} = 1.5$ [kgm^2], $C_{s1} = C_{s2} = 0.5$ [Nms], $r_s = 0.1$ [m], $b_s = 0.3$ [m]. The control parameters of the B_{s2} and K_1, K_2 are designed

$$B_{s2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \; K_1 = 200, \; K_2 = 20 \quad (37)$$

and parameter of scattering transformation is $b = 500$. It is assumed that the constant time delay $T = 0.02$ [s] exists. The initial condition are given as follows

$$q_m(0) = q_s(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \omega_m(0) = \omega_s(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (38)$$

Consider the control of the wheeled mobile robot shown in Fig. 4. The operator steers the wheeled mobile robot through the virtual image robot. First, we examine whether the wheeled mobile robot moves along the virtual image robot. Next, it is verified that the operator can feel the force from the obstacle, when the wheeled mobile robot hits the obstacle.

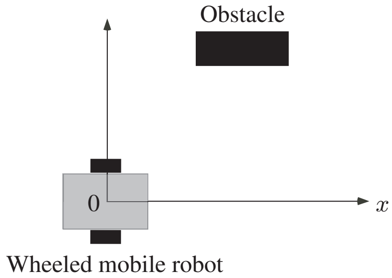

Fig. 4. Simulation Setup

The simulation results are obtained by using MATLAB. The simulation results show that the human operator gives the force as $F_{h1} = 5$ [Nm]$(t = 1$ [s]$- 11$ [s]$)$, $F_{h2} = 4$ [Nm]$(t = 1$ [s]$- 11$ [s]$)$, the slave mobile robot contacts the environment $F_e = 5$ [Nm]$(t = 50$ [s]$- 55$ [s]$)$ (Fig. 5). Figs. 6-8 indicates the position and rotation of master virtual image robot and slave wheeled mobile robot, where the solid line and dashed line represents the master virtual image robot and slave wheeled mobile robot, respectively. The trajectories of $x_m - y_m$ and $y_m - y_s$ are shown in Fig. 9. It is indicated that the error of the position and rotation $q_m(t-T) - q_s(t)$ is bounded. Figs. 10-11 shows the angle velocities ω_m, ω_s asymptotically converge to zero. Fig. 12 indicates the operator receives the force, when the wheeled mobile robot gives the force F_e. From the input torque F_{feed} shown in Fig. 13, the wheeled mobile robot moves along the virtual image robot.

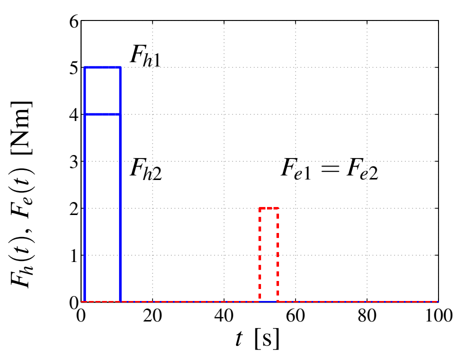

Fig. 5. Time responses of F_h and F_e (solid: F_h, dashed: F_e)

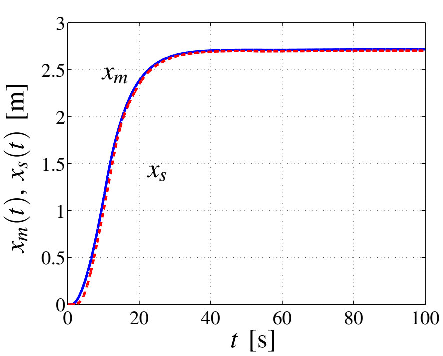

Fig. 6. Time responses of x_m and x_s (solid: x_m, dashed: x_s)

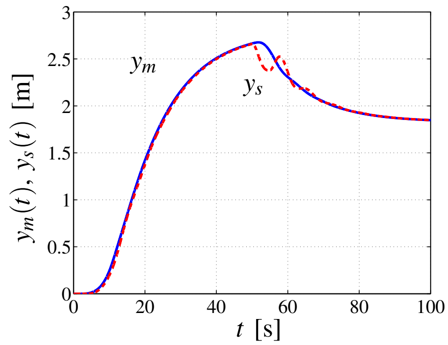

Fig. 7. Time responses of y_m and y_s (solid: y_m, dashed: y_s)

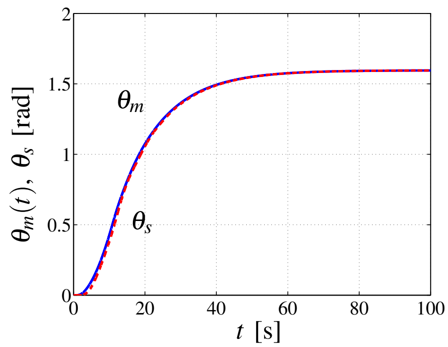

Fig. 8. Time responses of θ_m and θ_s (solid: θ_m, dashed: θ_s)

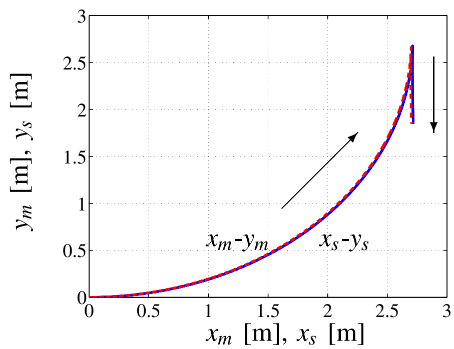

Fig. 9. Trajectories of x_m-y_m and x_s-y_s (solid: x_m-y_m, dashed: x_s-y_s)

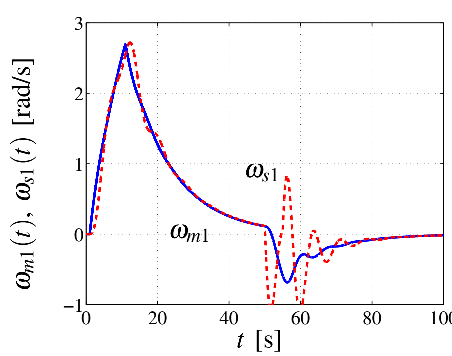

Fig. 10. Time responses of ω_{m1} and ω_{s1} (solid: ω_{m1}, dashed: ω_{s1})

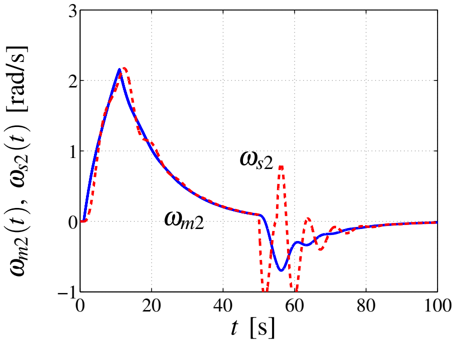

Fig. 11. Time responses of ω_{m2} and ω_{s2} (solid: ω_{m2}, dashed: ω_{s2})

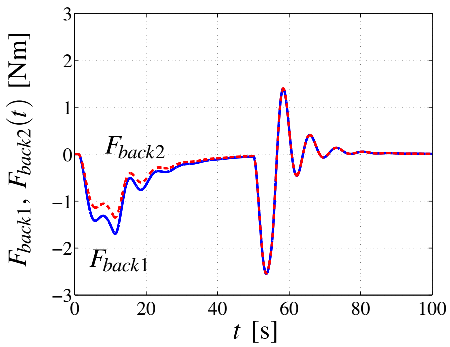

Fig. 12. Time responses of F_{back1} and F_{back2} (solid: F_{back1}, dashed: F_{back2})

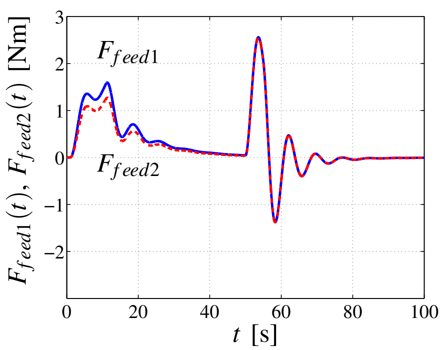

Fig. 13. Time responses of F_{feed1} and F_{feed2} (solid: F_{feed1}, dashed: F_{feed2})

V. CONCLUSION

This paper considered the bilateral teleoperation of the slave wheeled mobile robot with time delay using the master virtual robot model. The virtual robot model as a master robot was introduced. The human operator commands the virtual robot, the wheeled mobile robot tracks the motion of the virtual robot. The kinematics and dynamics of master/slave robots can be considered. The passivity based control schemes for bilateral teleoperation was applied, the control law was proposed. The matrix inequality to solve design parameters was derived. It was shown that the master and slave angle velocities asymptotically converge to the origin, the position tracking error remains bounded. In the simulation, the performance of the proposed bilateral teleoperation system was verified using wheeled mobile robots.

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