

A Sensor Scheduling Considering Estimation Error Variance and Communication Energy

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Abstract—This paper deals with a sensor scheduling considering estimation error variance and communication energy in sensor networked feedback system. We propose an decentralized estimation algorithm with unknown inputs in each sensor node. Most existing works deal with the sensor network systems as sensing systems and it is difficult to apply them to the real physical feedback control systems. Then some local estimates are merged and the merged estimates can be optimized in the proposed method and the estimation error covariance has a unique positive definite solution under some assumptions. Next, we propose a novel sensor scheduling algorithm in which each sensor node transmits information. A sensor node uses energy by communication between other sensor node or the plant. The proposed algorithm achieves a sub-optimal network topology with minimum energy and a desired variance. Experimental results show an effectiveness of the proposed method.

I. INTRODUCTION

Recently, much attention has been attracted to a wireless sensor network. It generally consists many sensor nodes with memory units, communications and calculation capabilities [1], [2]. In these researches, sensor nodes are connected wirelessly and some local estimates are merged into the common estimate via the wireless communication paths. It is well known that sensor networks are superior to an observation by a system with a single sensor in a fault tolerance, load reduction of operator, collection and application of information. Owing to some advantages, it is possible to apply various fields such as guidance control systems, traffic control systems, nano-medicines and disaster countermeasures. Meanwhile, each sensor node uses electric power for a communication and calculations, but the sensor nodes are generally powered and driven by built-in batteries. Moreover it is difficult to change batteries frequently or charge by power cable because of the increase in costs. Therefore, it is important to utilize the energy efficiently to achieve an energy-saving system and prolong sensor nodes life. For this objective, the sensor scheduling, the optimization of the communication rate or the buffer length and decreasing communication distances by the multi-hop communication have been studied [3–5]. Consequently, in this paper, we discuss a sensor scheduling problem considering the estimation error variance and the communication energy in the sensor

networked feedback control system, one of the approach to this objective.

Distributed Kalman Filter in sensor networks has been studied in [7–9]. Each sensor node calculates the local estimate and the sensor network system generates the common estimate merged via the communication paths between sensor nodes. But, they dealt with a sensor network system as a sensing system and do not consider arbitrary control inputs applied to the plant. Thus, it is difficult to apply to the guidance control. Moreover, they do not consider the communication energy. A. Goldsmith et al. have proposed some methods to achieve a energy-saving system. For example, they have investigated the optimization problem of a sensing rate, the relation between the estimation gain and the energy efficiency, etc. [10], [11]. But, they dealt with the plant without the control input and the fixed network topology. Additionally, the estimation algorithm was the weighted averaging. In our framework, we discuss *Distributed Kalman Filter* and the network configuration. Thus, we can not apply these previous works. Meanwhile, the network configuration and the sensor scheduling algorithm considering an estimation error variance and communication energy were proposed in [4–6]. But, each sensor node has only a observation and communication capability and does not have a calculation capability. The fusion center calculates the estimate and transmits the control input to the plant. In our framework, each sensor node has the calculation, communication and observation functions and the control input is applied to the plant. Thus we can not apply these previous methods.

In this paper, we discuss a sensor scheduling problem considering the estimation error variance and communication energy in a sensor networked feedback control system. We first propose an estimation algorithm with unknown inputs of the plant in the sensor networked feedback control system. Each sensor node calculates the local estimate without information of the control input and transmits its information to the sensor node applying the control input to the plant. This sensor node calculates the common estimate and control input based on received information. Then we show that there is the unique positive definite solution to the discrete algebraic Ricatti equation in the error covariance update. Secondly, we propose a sensor scheduling algorithm considering estimation error variance and communication energy. This scheduling algorithm achieves a sub-optimal network topology with minimum energy and a desired error variance. Finally, we verify effectiveness of a sensor scheduling algorithm by experiments.

This paper is organized as follows. The feedback control

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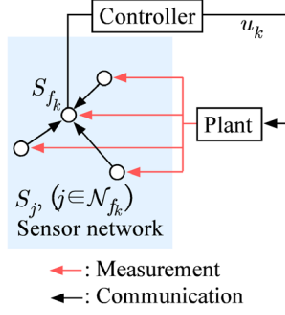


Fig. 1. Sensor network

system via a sensor network and the network topology are presented and problems are formulated accordingly in Section II. Section III describes a novel decentralized estimation algorithm with the unknown input and the unique solution to the discrete algebraic Ricatti equation under some assumptions. A sensor scheduling algorithm is proposed in Section IV. Finally, some experimental results are presented in Section V.

II. PROBLEM FORMULATION

A. Plant and Sensor Nodes

In this paper, we consider the sensor networked feedback control system illustrated in Fig. 1. This system consists of the plant and N sensor nodes S_i , ($i = 1, 2, \dots, N$). We assume all sensor nodes have enough computation capability and take a measurement of the plant. The process dynamics of the plant and the measurement equation of the sensor node S_i are given by

$$x_{k+1} = Ax_k + Bu_k + w_k, \quad (1)$$

$$y_k^i = C_i x_k + v_k^i, \quad (2)$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$, $y_k^i \in \mathbb{R}^{q_i}$ are the state, the control input and the measurement output of the sensor node S_i respectively. Additionally, $w_k \in \mathbb{R}^n$, $v_k^i \in \mathbb{R}^{q_i}$ are the process noise and the measurement noise respectively. We assume that the control input u_k is applied from the sensor node S_{f_k} , ($f_k = 1, 2, \dots, N$) to the plant and given by

$$u_k = L \hat{x}_k^{f_k}, \quad (3)$$

where $\hat{x}_k^{f_k} \in \mathbb{R}^n$ is the estimate of the sensor node S_{f_k} and L is the feedback gain. Now we assume we can arbitrarily determine which sensor node is the sensor node S_{f_k} at each time step. Thus, the task of the sensor node S_{f_k} is similar to the fusion center discussed in previous work, but it is not fixed. Moreover, (1) and (2) satisfy following assumptions 1-3.

Assumption 1: $w_k, v_k = [(v_k^1)^T \ (v_k^2)^T \ \dots \ (v_k^N)^T]^T \in \mathbb{R}^q$, ($q = \sum_i^N q_i$) are zero mean white Gaussian noise and

satisfy the following equations

$$E \left\{ \begin{bmatrix} w_k \\ v_k \end{bmatrix} \begin{bmatrix} w_k^T & v_k^T \end{bmatrix} \right\} = \begin{bmatrix} Q & \mathbf{0} \\ \mathbf{0} & R \end{bmatrix}, \quad (4)$$

$$E \{ w_k x_0^T \} = E \{ v_k x_0^T \} = \mathbf{0}, \quad (5)$$

where $Q, R = \text{diag}(R_1, R_2, \dots)$ are the positive semidefinite and positive definite covariance matrices of noises w_k, v_k respectively.

Assumption 2: The matrix pair $(A, Q^{\frac{1}{2}})$ is reachable.

Assumption 3: The matrix pair (C, A) is detectable, where

$$C = \begin{bmatrix} C_1^T & C_2^T & \dots & C_N^T \end{bmatrix}^T. \quad (6)$$

B. Network Topology

The sensor network consists N sensor nodes and one of them is the sensor node S_{f_k} applying the control input to the plant. We assume the sensor node S_{f_k} can communicate with other sensor nodes directory and define the set $\mathcal{N}_{f_k}$ containing sensor nodes communicating the sensor node S_{f_k} . Here there is no communication in between arbitrary two sensor nodes belonging to the set $\mathcal{N}_{f_k}$ at time step k . We assume we can arbitrary determine sensor nodes belonging to the set $\mathcal{N}_{f_k}$ as a case of the sensor node S_{f_k} .

Remark 1: The wireless communication between the sensor node S_j , ($j \in \mathcal{N}_{f_k}$) and S_{f_k} means that the sensor node S_j , ($j \in \mathcal{N}_{f_k}$) transmits information to the sensor node S_{f_k} . Thus, all communication paths are unidirectional.

In general, if there are the bidirectional communication paths, each sensor node can get and use a lot of information. But, in this paper, the network topology vary with time because we discuss a sensor scheduling problem determining the sensor node S_{f_k} and the set $\mathcal{N}_{f_k}$ each time step. Due to different communication ranges of each sensor node or obstacles, it is difficult to keep bidirectional communication path at all times in real physical system. Moreover, it can cause high machinery costs. Thus, we deals with the unidirectional communication path. Consequently, all sensor nodes satisfy following *Assumption 4*.

Assumption 4: The sensor node S_j , ($j \in \mathcal{N}_{f_k}$) can transmit to the sensor node S_{f_k} once while one time step with a time delay less than a sampling time. Additionally, when the sensor node S_{f_k} applies the control input u_k to the plant and sensor node S_j , ($j \in \mathcal{N}_{f_k}$) transmits information to the sensor node S_{f_k} . These sensor nodes use the communication energy $E_{f_k,p}, E_{j,f_k} \in \mathbb{R}_+$ respectively.

We define the total communication energy E_k of the system. The energy E_k is described as follows

$$E_k = E_{f_k, x_k} + \sum_{j \in \mathcal{N}_{f_k}} E_{j, f_k}. \quad (7)$$

Remark 2: The communication energy $E_{i,j}$ generally can be $E_{i,j} = b_{i,j} + a_{i,j}(d_{i,j})^{c_{i,j}}$ and depend on a distance between sensor nodes S_i and S_j , where $b_{i,j}$ is a static part and $a_{i,j}$ is a dynamic part. $c_{i,j}$ is typically from 2 through 6 [6].

C. Control Problems

In this paper, we discuss the estimation problem with unknown input u_k and a sensor scheduling problem. Problems can be formulated as following *problems 1, 2*.

Problem 1: We assume the plant and all sensor nodes satisfy *Assumptions 1-4* and the sensor node S_{f_k} and the set $\mathcal{N}_{f_k}$ is determined. Then compute the optimal state estimate $\hat{x}_k^{f_k}$ that minimizes the following estimation error variance.

$$J = E \left\{ (x_k - \hat{x}_k^{f_k})^T (x_k - \hat{x}_k^{f_k}) \right\}. \quad (8)$$

Problem 2: At time step k , find the optimal network topology T_k^* satisfying $J \leq \gamma$ and the following equation.

$$T_k^* = \arg \min_{T_k} E_k, \quad (9)$$

where $\gamma > 0$ is a design parameter.

III. ESTIMATION ALGORITHM

In this section, we propose the estimation algorithm in the sensor networked feedback control system. The proposed algorithm based on extension of *Decentralized Kalman Filter* in [9]. Each sensor node S_j , ($j \in \mathcal{N}_{f_k}$) computes the local estimate $\hat{x}_k^j$. Here these sensor nodes can not know the control input because all communication paths are unidirectional. We can not apply an existing method to the feedback system via a sensor network. Consequently, we propose the novel estimation algorithm considering the unknown control input. In this algorithm, each sensor node S_j , ($j \in \mathcal{N}_{f_k}$) transmits $\hat{x}_k^{j-}$, $\hat{x}_k^j$, P_k^{j-} , P_k^j to the sensor node S_{f_k} . The sensor node S_{f_k} computes estimate $\hat{x}_k^{f_k}$ by information from sensor nodes S_j , ($j \in \mathcal{N}_{f_k}$).

A. Estimation Algorithm of sensor nodes S_j , ($j \in \mathcal{N}_{f_k}$)

Firstly, we discuss an estimation algorithm of sensor nodes S_j , ($j \in \mathcal{N}_{f_k}$). Each sensor node S_j , ($j \in \mathcal{N}_{f_k}$) do not have information of the control input because all communication paths are unidirectional. Proposed algorithm satisfies the following *Theorem 1*.

Theorem 1: Consider the system (1) and (2) with *Assumption 1-4*. Then an estimation algorithm of each sensor node S_j , ($j \in \mathcal{N}_{f_k}$) is given by the following equations and the estimate $\hat{x}_k^j$ is the minimum variance estimate based measurements of sensor node S_j .

$$\hat{x}_{k+1}^{j-} = A\hat{x}_k^j + B\hat{u}_k^j, \quad (10)$$

$$\hat{x}_k^j = \hat{x}_k^{j-} + K_k^j(y_k^j - C_j\hat{x}_k^{j-}), \quad (11)$$

$$\hat{u}_k^j = L\hat{x}_k^j, \quad (12)$$

$$P_{k+1}^{j-} = (A + BL)P_k^j(A + BL)^T + Q + BLP_k^{f_k}L^TB^T - (A + BL)M_k^jL^TB^T - BL(M_k^j)^T(A + BL)^T, \quad (13)$$

$$P_k^j = \left\{ (P_k^{j-})^{-1} + C_j^TR_j^{-1}C_j \right\}^{-1}, \quad (14)$$

$$M_k^j = (I - K_k^jC_j)M_k^{j-}(I - K_k^{f_k}C_{f_k})^T, \quad (15)$$

$$M_{k+1}^{j-} = (A + BL)M_k^jA^T + Q - BLP_k^{f_k}A^T, \quad (16)$$

where definition of each variable is described as follows

$$\hat{x}_k^{j-} = E \left\{ x_k | y_{k-1}^j, y_{k-2}^j, \dots \right\},$$

$$\hat{x}_k^j = E \left\{ x_k | y_k^j, y_{k-1}^j, \dots \right\},$$

$$P_k^{j-} = E \left\{ (x_k - \hat{x}_k^{j-})(x_k - \hat{x}_k^{j-})^T \right\},$$

$$P_k^j = E \left\{ (x_k - \hat{x}_k^j)(x_k - \hat{x}_k^j)^T \right\},$$

$$M_k^j = E \left\{ (x_k - \hat{x}_k^j)(x_k - \hat{x}_k^{f_k})^T \right\},$$

$$M_k^{j-} = E \left\{ (x_k - \hat{x}_k^{j-})(x_k - \hat{x}_k^{f_k})^T \right\}.$$

Proof: The filter equation for (1) and (2) are given by

$$\hat{x}_{k+1}^{j-} = A\hat{x}_k^j + B\hat{u}_k^j, \quad (17)$$

$$\hat{x}_k^j = \hat{x}_k^{j-} + K_k^j(y_k^j - C_j\hat{x}_k^{j-}), \quad (18)$$

$$\hat{u}_k^j = L\hat{x}_k^j. \quad (19)$$

From (1)-(3), (17), (18) and (19), errors $e_k^j = x_k - \hat{x}_k^j$, $e_k^{j-} = x_k - \hat{x}_k^{j-}$ can be described as follows

$$e_k^j = (I - K_k^jC_j)e_k^{j-} - K_k^jv_k^j, \quad (20)$$

$$e_{k+1}^{j-} = (A + BL)e_k^j + w_k - BLv_k^{f_k}. \quad (21)$$

Thus estimation error covariance matrices P_k^j and P_{k+1}^{j-} are described as follows

$$P_k^j = (I - K_k^jC_j)P_k^{j-}(I - K_k^jC_j)^T + K_k^jR_j(K_k^j)^T, \quad (22)$$

$$P_{k+1}^{j-} = (A + BL)P_k^j(A + BL)^T + Q + BLP_k^{f_k}L^TB^T - (A + BL)M_k^jL^TB^T - BL(M_k^j)^T(A + BL)^T, \quad (23)$$

where M_k^j is the cross covariance matrix between the estimation errors of the estimate $\hat{x}_k^{f_k}$ and $\hat{x}_k^j$.

Firstly, we consider the covariance matrix (22). From the condition $\frac{\partial}{\partial K_k^j} \text{tr} P_k^j = \mathbf{0}$ and (22), the filter gain K_k^j and error covariance P_k^j can be described as follows

$$K_k^j = P_k^j C_j^T R_j^{-1}. \quad (24)$$

$$P_k^j = \left\{ (P_k^{j-})^{-1} + C_j^T R_j^{-1} C_j \right\}^{-1}. \quad (25)$$

Secondly, we consider the cross covariance matrix M_k^j in (23). From its definition, M_k^j is described as follows

$$M_k^j = (I - K_k^jC_j)M_k^{j-}(I - K_k^{f_k}C_{f_k})^T. \quad (26)$$

The sensor node S_{f_k} knows the value of the control input u_k because this sensor node applies the control input to the plant. Thus the estimation error $e_k^{f_k}$ is given as the following

$$e_{k+1}^{f_k} = Ae_k^{f_k} + w_k. \quad (27)$$

From (27) and its definition, the cross covariance matrix M_k^{j-} is given by

$$M_{k+1}^{j-} = (A + BL)M_k^jA^T + Q - BLP_k^{f_k}A^T. \quad (28)$$

■

Next, we consider the estimation algorithm of the sensor node S_{f_k} . The estimation of the sensor node S_{f_k} is based on its measurement and the received information $\hat{x}_k^{j-}$, $\hat{x}_k^j$, P_k^j and P_k^j from some sensor nodes S_j , ($j \in \mathcal{N}_{f_k}$). The sensor node S_{f_k} has information of the control input u_k . Thus, the estimation algorithm of the sensor node S_{f_k} is following *Decentralized Kalman Filter* proposed in [9].

$$\hat{x}_{k+1}^{f_k-} = (A + BL)\hat{x}_k^{f_k-}, \quad (29)$$

$$\bar{x}_k^{f_k} = \hat{x}_k^{f_k-} + K_k^{f_k}(y_k^{f_k} - C_{f_k}\hat{x}_k^{f_k-}), \quad (30)$$

$$K_k^{f_k} = \bar{P}_k^{f_k} C_{f_k}^T R_{f_k}^{-1}, \quad (31)$$

$$P_k^{f_k-} = A P_k^{f_k} A^T + Q, \quad (32)$$

$$\bar{P}_k^{f_k} = \left\{ (P_k^{f_k-})^{-1} + C_{f_k}^T R_{f_k}^{-1} C_{f_k} \right\}^{-1}, \quad (33)$$

$$P_k^{f_k} = \left[(\bar{P}_k^{f_k})^{-1} + \sum_{j \in \mathcal{N}_{f_k}} \left\{ (P_k^j)^{-1} - (P_k^{j-})^{-1} \right\} \right]^{-1}, \quad (34)$$

$$\hat{x}_k^{f_k} = P_k^{f_k} \left[(\bar{P}_k^{f_k})^{-1} \bar{x}_k^{f_k} + \sum_{j \in \mathcal{N}_{f_k}} \left\{ (P_k^j)^{-1} \bar{x}_k^j - (P_k^{j-})^{-1} \hat{x}_k^{j-} \right\} \right], \quad (35)$$

where the definition of variables is as follows

$$\begin{aligned} \bar{x}_k^{f_k} &= E \left\{ x_k | y_k^{f_k}, y_{k-1}^{f_k}, \dots \right\}, \\ \hat{x}_k^{f_k} &= E \left\{ x_k | y_k^{f_k}, y_{k-1}^{f_k}, \dots, y_k^j, y_{k-1}^j \right\}, j \in \mathcal{N}_{f_k}, \\ \bar{P}_k^{f_k} &= E \left\{ (x_k - \bar{x}_k^{f_k})(x_k - \bar{x}_k^{f_k})^T \right\}, \\ P_k^{f_k} &= E \left\{ (x_k - \hat{x}_k^{f_k})(x_k - \hat{x}_k^{f_k})^T \right\}. \end{aligned}$$

The estimate $\bar{x}_k^{f_k}$ is only based on measurements of the sensor node S_{f_k} . But, the estimate $\hat{x}_k^{f_k}$ is based on measurements of the sensor node S_{f_k} and sensor nodes belong to the set $\mathcal{N}_{f_k}$. Then the covariance matrix $P_k^{f_k}$ satisfies the following *Theorem 2*.

Theorem 2: Consider the system (1) and (2) with *Assumptions 1-4*. If sensor nodes $S_{f_k} = S_f, S_{j_1}, S_{j_2}, \dots$, ($j_1, j_2 \in \mathcal{N}_f$) are determined and the matrix pair (H_f, A) , $H_f = [C_f^T \ C_{j_1}^T \ C_{j_2}^T \ \dots]^T$ is detectable, then the estimate $\hat{x}_k^f$ is the solution of *Problem 1* and there is a unique positive definite solution P_∞^f of the following algebraic Ricatti equation.

$$P_\infty^f = A P_\infty^f A^T + Q - A P_\infty^f H_f^T (H_f P_\infty^f H_f^T + V_f)^{-1} H_f P_\infty^f A^T, \quad (36)$$

where $V_f = \text{diag}\{R_f, R_{j_1}, R_{j_2}, \dots\}$.

Proof: Substituting (14) into (34), we can get

$$P_k^f = \left[\left(P_k^{f-} \right)^{-1} + H_f^T V_f^{-1} H_f \right]^{-1} \quad (37)$$

From (32) and (37), this is the algebraic Ricatti equation. Consequently, From *Assumption 2* and detectability of the

matrix pair (H_f, A) , the covariance matrix P_k^f has the unique positive definite solution P_∞^f . ■

From *Theorem 2*, there is the unique positive definite solution of the algebraic Ricatti equation (32)-(34) while sensor nodes S_{f_k} and S_j , ($j \in \mathcal{N}_{f_k}$) are determined. Additionally, from *Assumption 3*, if we use $N - 1$ sensor nodes as S_j , ($j \in \mathcal{N}_{f_k}$), there is the unique positive definite solution of the algebraic Ricatti equation. In next section, we propose a sensor scheduling algorithm considering the estimation error variance $J = \text{tr} P_\infty^{f_k}$ and the communication energy. If we determine the set $\mathcal{N}_{f_k}$ including all sensor nodes, the estimation error variance of the common estimate is minimized. But the communication energy will increase because all sensor nodes have to transmit information to the sensor node S_{f_k} . On the contrary, if we determine the set $\mathcal{N}_{f_k}$ is empty set, the communication energy is zero because there are no communication paths. But the estimation error variance of the estimate will increase. Consequently, there is a trade-off between the estimation accuracy and the communication energy.

IV. SENSOR SCHEDULING ALGORITHM

In previous section, we showed that the estimation error variance of the estimate $\hat{x}_k^{f_k}$ can be written as $J = \text{tr}(P_\infty^{f_k})$. In this section, we propose a sensor scheduling algorithm minimizing communication energy in subset of all available network topology under the condition $J \leq \gamma$. The network topology can be fixed uniquely if and only if we determine the sensor nodes S_{f_k} and S_j , $j \in \mathcal{N}_{f_k}$. Here we can get that $N 2^{N-1}$ network topologies are available. Consequently, we propose the following algorithm to reduce computation costs. In the proposed algorithm, $N(N-1)$ network topologies are available. Additionally, $E(S_i, \mathcal{N}_i)$ and $J(S_i, \mathcal{N}_i)$ are communication energy of the whole system and the estimation error variance respectively when sensor node $S_{f_k} = S_i$ and the set $\mathcal{N}_i$ are determined.

Sensor Scheduling Algorithm

- 1: **for** $\alpha = 1$ to N **do**
- 2: $\mathcal{N}_\alpha = \mathcal{B} = \{1, \dots, N\} \setminus \alpha$
- 3: **repeat** $N - 1$
- 4: $\beta = \arg \max_{j \in \mathcal{N}_\alpha \cap \mathcal{B}} E_{\alpha, j}$
- 5: **if** $J(S_\alpha, \mathcal{N}_\alpha \setminus S_\beta) \leq \gamma$ **then**
 $\mathcal{N}_\alpha := \mathcal{N}_\alpha \setminus S_\beta$
- 6: $\mathcal{B} := \mathcal{B} \setminus S_\beta$
- 7: **return** $S_{i^*}, \mathcal{N}_{i^*}$, ($i^* = \min_{i=1, \dots, N} E(S_i, \mathcal{N}_i)$)

In this algorithm, firstly, we determine the sensor node $S_{f_k} = S_\alpha$, ($\alpha = 1$). Secondly, we remove the sensor node S_β from the set $\mathcal{N}_\alpha$ in order of decreasing the communication energy $E_{\alpha, \beta}$ under the condition $J(S_\alpha, \mathcal{N}_\alpha \setminus S_\beta) \leq \gamma$. We calculate these subroutine N times ($\alpha = 1, 2, \dots, N$). Finally, the sensor node S_{f_k} and the set $\mathcal{N}_{f_k}$ minimizing communication energy in subset of all available network topology under the condition $J \leq \gamma$ are determined.

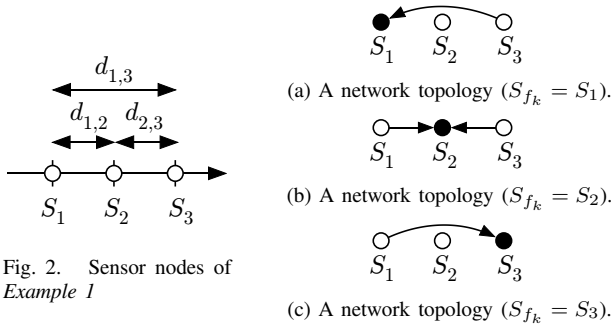
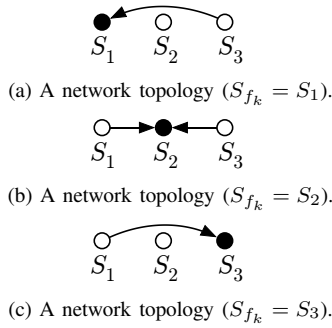


Fig. 3. Network topologies of Example 1



Example 1 is described as follows.

Example 1: Consider 3 sensor nodes ($N = 3$) illustrated in Fig. 2. We assume the following conditions.

- 1) the Distances are $d_{1,2} = d_{2,3} = 1$, $d_{1,3} = 2$.
- 2) A communication energy is $E_{i,j} = \epsilon d_{i,j}^2$, ($\epsilon > 0$).
- 3) The condition $J \leq \gamma$ is satisfied if and only if we use sensor nodes (S_1, S_2, S_3) or (S_1, S_3) .

Now, we examine the proposed sensor scheduling algorithm in Example 1.

We first define $\alpha = 1$ and $\mathcal{N}_1 = \mathcal{B} = \{2, 3\}$. These mean that we first check the communication energy in a case of the sensor node S_{f_k} is S_1 . Then 4., 5. and 6. in a sensor scheduling algorithm are calculated 2 times. we can chose $\beta = 3$ at the initial calculation. Then the sensor node S_3 would not be removed from $\mathcal{N}_\alpha$ because the condition $J(S_\alpha, \mathcal{N}_\alpha \setminus S_\beta = \{2\}) \leq \gamma$ is not satisfied. Consequently, $\mathcal{N}_\alpha = \{2, 3\}$, $\mathcal{B} = \{2\}$. After the initial calculation, we can chose $\beta = 2$ at the second calculation. Because the condition $J(S_\alpha, \mathcal{N}_\alpha \setminus S_\beta = \{3\}) \leq \gamma$ is satisfied, the sensor node S_2 is removed. Consequently, if we determine the sensor nodes S_{f_k} is S_1 , the set $\mathcal{N}_1 = \{3\}$ (see Fig. 3(a)) and communication energy E_k is given by

$$E(S_1, \mathcal{N}_1) = E_{1,x_k} + \epsilon d_{1,3}^2 = E_{1,x_k} + 4\epsilon. \quad (38)$$

Next, we can define $\alpha = 2$ and $\mathcal{N}_\alpha = \mathcal{B} = \{1, 3\}$. We can calculate the communication energy E^2 and the set $\mathcal{N}_2$ by a method similar to above calculation. In this subroutine, because we can not remove sensor nodes from the set $\mathcal{N}_2$ under the condition $J_2 \leq \gamma$, we can define $\mathcal{N}_2 = \{1, 3\}$ (see Fig. 3(b)) and the communication energy is given by the following equation when the sensor node S_{f_k} is S_2

$$E(S_2, \mathcal{N}_2) = E_{2,x_k} + \epsilon (d_{1,2}^2 + d_{2,3}^2) = E_{2,x_k} + 2\epsilon. \quad (39)$$

Finally we choose $\alpha = 3$ and $\mathcal{N}_3 = \{1, 2\}$. Then we can remove the sensor node S_2 from the set $\mathcal{N}_3$ under the condition. Consequently, $\mathcal{N}_\alpha = \{3\}$ (see Fig. 3(c)) and the communication energy is calculated as the following equation.

$$E(S_3, \mathcal{N}_3) = E_{3,x_k} + \epsilon d_{1,3}^2 = E_{3,x_k} + 4\epsilon \quad (40)$$

(38)-(40) are the communication energies when the sensor nodes S_{f_k} is S_1 , S_2 or S_3 respectively. We consider the energy to transmit information from each sensor node to the

plant is $E_{1,x_k} = \epsilon$, $E_{1,x_k} = 4\epsilon$, $E_{1,x_k} = 9\epsilon$. at time step k . Then the communication energy are given as follows

$$E(S_1, \mathcal{N}_1) = 5\epsilon, \quad E(S_2, \mathcal{N}_2) = 6\epsilon, \quad E(S_3, \mathcal{N}_3) = 13\epsilon. \quad (41)$$

Consequently, we can determine $S_{i^*} = S_1$, $\mathcal{N}_{i^*} = \{3\}$ at time step k .

V. EXPERIMENTAL EVALUATION

In this section, an effectiveness of a sensor scheduling algorithm is evaluated by experiments. The experiment was carried out on the two-wheeled vehicle, the CCD camera and the computer as shown in Fig. 4. All measurement outputs are calculated from the image of the CCD camera mounted above the vehicle. The video signals are acquired by a frame grabber board PicPort-color and image processing software HALCON generate nine measurements. Consequently, nine sensor nodes, a network topology and measurement noises exist in the computer. We use DS1104 (dSPACE Inc.) as a real-time calculating for the estimation and sensor scheduling. Here Two-wheeled vehicle has the nonholonomic constraint. But the two-wheeled vehicle can be treated as a linear plant as the following equations by virtual structure for a feedback linearization [13].

$$A = \begin{bmatrix} 1 & 0 & \delta & 0 \\ 0 & 1 & 0 & \delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{\delta^2}{2} & 0 \\ 0 & \frac{\delta^2}{2} \\ \delta & 0 \\ 0 & \delta \end{bmatrix},$$

where $\delta = 0.2$ and $x_0 = [1.3 \ 0.7 \ 0 \ 0]^T$ are the sampling time and the initial state respectively. We design the feedback gain L by LQG control. We assume nine sensor nodes are available and each sensor node has the following measurement equation and these positions are shown in Fig. 5.

$$\begin{aligned} y_k^i &= [1 \ 0 \ 0 \ 0] x_k + v_k^i, & (i = 1, 5, 9) \\ y_k^i &= [0 \ 1 \ 0 \ 0] x_k + v_k^i, & (i = 2, 6) \\ y_k^i &= [0 \ 0 \ 1 \ 0] x_k + v_k^i, & (i = 3, 7) \\ y_k^i &= [0 \ 0 \ 0 \ 1] x_k + v_k^i, & (i = 4, 8) \end{aligned}$$

Additionally, the covariance matrices of noises are $Q = 1 \times 10^{-4} I_4$, $R = 0.1 I_9$ respectively.

We define the communication energy between arbitrarily two sensor nodes. We assume that the communication energy between sensor nodes S_i and S_{f_k} is $E_{i,f_k} = \epsilon d_{i,f_k}^2$. Here, d_{i,f_k} is the distance between sensor nodes S_i and S_{f_k} and ϵ is a positive constant.

The experiment was done by choosing $\gamma = 0.02$. The experimental results are shown in Fig. 6. Fig. 6(a)-(c) show the trajectory of the vehicle and the network topology. As shown in Fig. 6(a)-(c), the sensor nodes are switched while the vehicle is moving. Fig. 6(b) shows the estimation error. As shown in Fig. 6(b), the estimation error is zero mean. Fig. 6(e) shows the estimation error variance $P_k^{f_k}$. As shown in Fig. 6(e), the estimation error variance converge to the solution of the algebraic Ricatti equation and the solution is less than design parameter γ at all times. Finally, Fig. 6(e) is a comparison between the following Cases 1, 2.

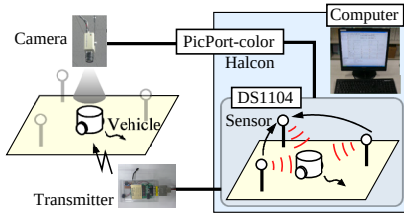


Fig. 4. Experimental setup

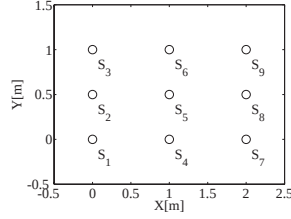


Fig. 5. Position of sensor nodes

Case 1: A sensor scheduling algorithm is applied.

Case 2: S_6 is the sensor node S_{f_k} at all times.

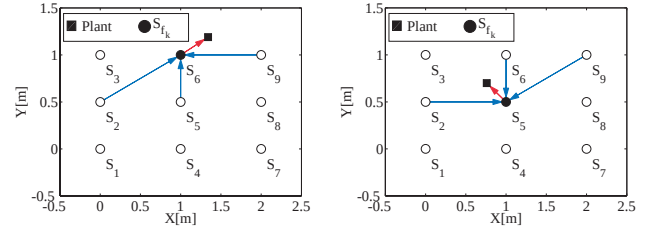
In these case, the error variance $\text{tr}P_k^{f_k}$ is same. But from Fig. 6(f) the communication energy is different. This figure shows the communication energy of the whole system is reduced by a sensor scheduling algorithm. Consequently, by designing γ , the proposed algorithm reduce the communication energy under the condition $J \leq \gamma$.

VI. CONCLUSIONS

In this paper, we discussed a sensor scheduling problem considering the estimation error variance and the communication energy in the sensor networked feedback control system. We first have proposed the estimation algorithm with the unknown input of the plant in the feedback control system via a sensor network. Each sensor node calculates the local estimate without information of the control input. After the calculation, it transmits information of the local estimate to the sensor node applying the control input to the plant. This sensor node calculates the common estimate and control input based on received information. Then we showed that there is the unique positive definite solution of the discrete algebraic Ricatti equation in the error covariance update. Secondly, we have proposed a sensor scheduling algorithm considering the estimation error variance and the communication energy. This scheduling algorithm achieved a sub-optimal network topology with the minimum energy and a desired error variance. Finally, we have verified the effectiveness of the proposed sensor scheduling algorithm by the experiments.

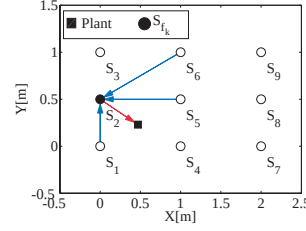
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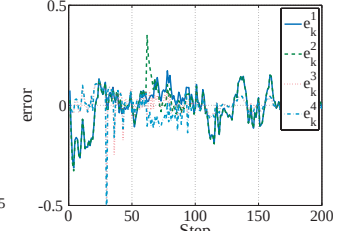


(a) Network structure at $k = 0$.

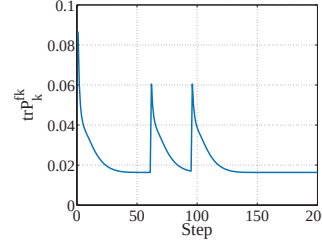
(b) Network structure at $k = 61$.



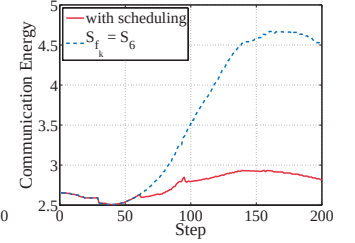
(c) Network structure at $k = 96$.



(d) Estimation error $e_k = x_k - \hat{x}_k$.



(e) Variance $J = \text{tr}P_k^{f_k}$.



(f) Communication energy E_k .

Fig. 6. Experimental results

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