Robot Localization and Mapping Problem with Unknown Noise Characteristics

Hamzah Ahmad and Toru Namerikawa

***Abstract*— In this paper, we examine the H_∞ filter-based SLAM especially about its convergence properties. In contrast to Kalman filter approach that considers gaussian noise with zero mean, H_∞ filter is more robust and may provide sufficient solutions for SLAM in an environment with unknown statistical behavior. Due to this advantage, H_∞ filter is proposed in this paper to efficiently estimate the robot and landmarks location under worst case situations. H_∞ filter requires the designer to appropriately choose the noise's covariance with respect to γ to obtain a desired outcome. We show some of the conditions to be satisfy in order to achieve better estimation results than Kalman filter. From the experimental results, H_∞ filter is perform better than Kalman filter for a case of bigger robot initial uncertainties. These subsequently may provide another available estimation method with the capability to ensure and improve estimation for the robotic mapping problem, especially in SLAM.**

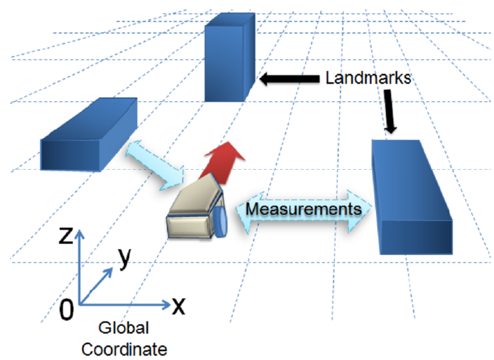

Fig. 1. Illustration for SLAM problem

I. INTRODUCTION

A. Robotic Mapping

Robotics localization and mapping problem is one the area of autonomous robot application that recently gained researcher's attention thanks to its capability that able to support fully autonomous robot behavior. The problem illustrates a case where a mobile robot is put in an unknown environment, then takes sufficient observations of its surroundings. Next, from this information, robot then builds a map from what it believes. Even though the development of the robot localization and mapping problem has passed about two decades, there are still a lot of difficulties to be solved.

Since 1990's, researchers around the world become more enthusiastic about this problem and a series of influential seminal papers by Smith and Cheeseman et.al [1], has boosted up this research and consequently evolved its name to Simultaneous Localization and Mapping problem(SLAM)[2]. See Fig.1 for further explanation. As stated by its name, SLAM consists of two general problems that are the robot localization and mapping. Robot localization states a problem where we are given predefined landmarks, the robot must try to estimate it location. In the other hand, robot mapping determines a problem that given a robot trajectory, a map must be build. Therefore, SLAM more complicated and needs proper effort for the solution. Nowadays, SLAM has been applied in a variety of applications, indoor or outdoor such as satellite, mining, space exploration, rescue, military, etc. The development of SLAM continues whether in 2D [3] or 3D applications [4][5] and amazingly expand even to home-based robot application. This problem is tracked around 1980's, and improved from the form of *Topological* and *Metric* approach to *Behavioral approach*, *Mathematical-based model* approach and *Probabilistic approach* [2]. However, between these 3 techniques, the probabilistic approach made a significant success than the mathematical models approach; which require building a precise model, or the behavior approach; a method of exploiting the sensor's behavior to the system. In spite of remarkable achievement of probabilistic approach, it has shortcomings of computational complexity. Nevertheless, with modern software development, a considerable support and solution to this problem may be available, consequently, inspire the development of SLAM problem.

Recently, probabilistic approaches whether parametric or non-parametric methods have been proposed to solve the SLAM problems such as Kalman filter, Unscented Kalman filter, Particle filter, etc. At this end, a non-parametric method called Fast-SLAM approach [2], efficiently constructs the unknown map by utilizing an amount of particle whose behaves as the uncertainty. If more particles are used, the estimation will be better, but in contrast they require a high computational cost for the systems. Due to such deficiencies, such a remarkable technique does not deter some classical methods, for example, Kalman filter and other conventional methods. Moreover, no matter what kind of filters presented above, they are still familiar and fundamentally relied on probabilistic theory. The readers are encouraged to read about the development of SLAM in [6] which discussed the SLAM problem from various aspects.

H.Ahmad is with Div. of Electrical Engineering and Computer Science, Graduate School of Natural Science and Technology, Kanazawa University, Kakuma-Machi,920-1192, Ishikawa, Japan hamzah@ump..edu.my

T.Namerikawa is with the Department of System Design Engineering, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama, 223-8522, Japan namerikawa@sd.keio.ac.jp

B. Probabilistic-based SLAM

Uncertainties and sensor noises are the most influential keywords that brought the idea of probabilistic into SLAM problem. Governed by the law of probabilistic, the estimation is processed to a set of information than only relying on a single guessed method. This eventually made probabilistic method applicable to most SLAM problems in most situations with unknown noise characteristics. In view to realize the truly autonomous robot's characteristics, probabilistic approach is one of the available approaches as it is able to assign and feeds sufficient information for the robot to make a judgement while they work or operate independently in a less-human monitoring system.

In contrast to Kalman filter reputation among decades within various fields, some applications still demands further attention for development especially regarding its deficiencies of zero mean gaussian assumption. Therefore, it is a wise decision to model a system that takes into account for a worst case of noise or when the noise statistics are partially known. Hence, H_∞ filter is proposed in this paper to study its behavior in SLAM to tolerate with such a robust system. The development of H_∞ filter for SLAM problem is theoretically shown with a brief comparison with Kalman filter approach [7][8]. H_∞ filter has been introduced by Mike Grimble[9] and act as one of the set-membership approaches, which assumes that the noise is known in bounded energy. It is also a technique that assumed the systems are provided with a priori information for estimation [10][11]. H_∞ filter guarantees that the energy gain from the noise inputs to the estimation errors is less than a certain level.

Throughout this paper, we examine the Kalman filter and H_∞ filter performance in linear and nonlinear case SLAM problem. We investigate the results using a constant motion and sensors uncertainties with a perfect data association. Even though this is seems to be simplistic, it gives a feasible study about the estimation. H_∞ filter is still new in the robotic mapping problem solution schemes such as SLAM, although it has desirable properties and competitive compare to Kalman filter. Kalman filter and Extended Kalman filter(EKF) have been studied immensely towards the SLAM problem using various approaches such as in [12][13]. [14] reported that, EKF with robocentric local mapping approach, is able to decrease location uncertainty of each location. West et.al [15] proved H_∞ filter was competent with other well-known approaches such as Kalman filter and Particle filter for SLAM problem. However, they did not present any theoretical explanation or contribution about H_∞ filter properties.

This paper is organized as follows. In section II, SLAM preliminary model is presented. Section III describes a brief introduction of H_∞ filter with a comparison between H_∞ algorithm and the Kalman filter, while section IV demonstrates the main results of convergence properties of H_∞ SLAM. Section V provides experimental results of H_∞ SLAM problem. Section VI represents the experimental results of SLAM using both filters. Finally section 6, concludes the paper.

II. SLAM Preliminary Model

The robot kinematics model should be determined to understand the robot motion through the environment. This information is used to predict the robot trajectory through the environment. The landmarks or features are also important to explicitly represent the state of the environment. In our case, landmarks are assumed to be stationary for convenience. SLAM consists of two general model; Process Model and Measurement/Observation Model. Each of this model plays an important role to achieve better estimation about the landmarks and robot location. For the SLAM process model, we have the following. We consider linear SLAM as most of the calculation is linearized in entire process and may describe the whole system.

$$x_{R_{k+1}} = F_{R_k} x_{R_k} + u_{R_k} + v_{R_k}, \tag{1}$$

where F_{R_k} is the state transition matrix, x_{R_k}, is the robot state, u_{R_k} is a vector of control inputs, and v_{R_k} is a vector of temporally uncorrelated process noise errors with zero mean and covariance, Q_{R_k}. The location of the n^{th} landmark is denoted as p_n. For the stationary landmarks p, and for $i = 1 \dots N$ states of landmarks are expressed as

$$p_{n_{k+1}} = p_{n_k} = p_n \tag{2}$$

Using above notation with respect to [1], the augmented process model consists of robot and landmarks location is as following.

$$x_{k+1} = F_k x_k + u_k + v_k \tag{3}$$

On the other hand, the measurement model demonstrates information about relative distance and angle between the robot and any landmarks. The measurement of an observation at i_{th} specific landmark/feature, yields the following equation.

$$\begin{aligned} z_k &= H_k x_k + w_k & (4)\\ &= H_{p_i} p_i - H_{v_k} x_{(v_k)} + w_k & (5)\end{aligned}$$

where w_k is a vector of temporally uncorrelated observation errors with zero mean and variance R_k. H_k is the observation matrix and represent the output of the sensor z_k to the state vector x_k when observing the i^{th} landmark. H_{p_i} and H_{v_k} is the observation matrix for the landmarks and the robot respectively. Alternatively, the observation model for the i^{th} landmark is written in the form

$$H_i = [-H_v, 0 \dots 0, H_{pi}, 0 \dots 0] \tag{6}$$

Above equation means observations are taken as a relative measurement between vehicle and landmarks. Both models are use recursively to predict and update both landmarks and robot position. Based on the data obtained from these two models, then the robot builds a map. Same to Kalman's filter, H_∞ filter has the prediction and updates process. Details will be explained in the next section consisting of some basic assumption of noises and a brief comparison to the Kalman filter approach.

III. H_∞ FILTER-BASED SLAM

This section presents the development of H_∞ filter-Based SLAM by considering its convergence properties. Due to our approach is probabilistic SLAM, the state covariance matrix plays an important role to determine the level of confidence for estimation. In SLAM, small state covariance matrix is desired. Hence, the analysis is focusing on the convergence behavior of H_∞ filter-Based SLAM, whether it may surpass Kalman filter performance or else.

The comparability of H_∞ filter and Kalman filter for a stationary robot case observing landmarks is evaluated in the experiments. Some brief explanation and preparation are introduced regarding the differences between H_∞ filter and Kalman filter before getting in depth with the filter performance in SLAM. The papers in [7][9] presented a satisfactory explanation of the H_∞ filtering. Referring to those, we first assume for the noise to have the following statistic.

Assumption 1: $R \triangleq DD^T \geq 0$

The above assumption is used to define that the measurements are correlated with noise. We also assume that the noise is in bounded energy which also a characteristic of H_∞ filter. This is the main dissimilarity between H_∞ filter and Kalman filter.

Assumption 2: Bounded noise energy; $\sum_{t=0}^{N}\|w_k\|^2 < \infty, \sum_{t=0}^{N}\|v_k\|^2 < \infty$

$\Sigma_0 \geq 0$, $Q_k \geq 0$, and $R_k \geq 0$ are the weighting matrices for state x_k, noise w_k, and v_k respectively. Details of H_∞ filter, is included in [7].

The difference between Kalman filter and H_∞ filter exists in the form of gain and covariance characteristics for each prediction and updates process. For Kalman filter, the equation for its gain and covariance are given by,

$$K_k = P_k(I + H_k^T R_k^{-1} H_k P_k)^{-1} \quad (7)$$

$$P_{k+1} = F_k P_k (I + H_k^T R_k^{-1} H_k P_k)^{-1} F_k^T + Q_k \quad (8)$$

As for H_∞ filter, the equation for its gain and covariance are given by

$$K_k = P_k(I - \gamma^{-2} I P_k + H_k^T R_k^{-1} H_k P_k)^{-1} \quad (9)$$

$$P_{k+1} = F_k P_k (I - \gamma^{-2} I P_k + H_k^T R_k^{-1} H_k P_k)^{-1} F_k^T + Q_k \quad (10)$$

I is an identity matrix with an appropriate dimension. Stated above, H_∞ filter depends on the covariance matrix of error signals, Q_k, R_k which are chosen and designed to achieve desired performance and all of these parameters must be bigger than zero. It is observable that, if γ values become bigger, this equation will be the same as (7),(8) of Kalman filter.

IV. MAIN RESULTS

We begin the convergence analysis of H_∞ filter by presenting the filter algorithm as stated below. The solution of an H_∞ filtering problem is as following[7],

$$P_{k+1} = F_k P_k \psi_k^{-1} F_k^T + G_k Q_k G_k^T, \quad P_0 = \Sigma_0 \quad (11)$$

$$\psi_k = I + (H_k^T R_k^{-1} H_k - \gamma^{-2} I) P_k \quad (12)$$

where I is an identity matrix with an appropriate dimension. Equation (11), (12) holds a Positive Semidefinite(PsD) solution if it satisfies

$$\hat{P}_k^{-1} - \gamma^{-2} I \geq 0, \quad k = 0, 1, \ldots, N, \quad (13)$$

where

$$\hat{P}_k = (P_k^{-1} - H_k^T R_k^{-1} H_k) \geq 0 \quad (14)$$

For $\gamma > 0$, the suboptimal H_∞ filter is given by below equations.

$$\hat{x}_{k+1|k} = F_k \hat{x}_{k|k} \quad (15)$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k[y_k - H_k \hat{x}_{k|k-1}], \hat{x}_{0|-1} = \bar{x}_0 \quad (16)$$

$$K_k = P_k H_k (H_k P_k H_k^T + R_k)^{-1} \quad (17)$$

Assumption 3: (F, H) is observable and (F, G) is controllable.

Lemma 1: Equation (11) is a PsD matrix if and only if $R_k \leq \gamma^2$.

Proof: For convenience, a 1-D monobot observing one landmark case is considered. Given that the initial covariance matrix, P_0

$$P_0 = \begin{bmatrix} P_R & 0 \\ 0 & P_m \end{bmatrix} \quad (18)$$

where P_R is the monobot state covariance and P_m is a landmark state covariance. If $\gamma^2 \geq R_k$ then (12) exhibit a PsD matrix. This can be proven as follows. Note that for 1-D monobot case, the measurement matrix becomes $H = [-1 \quad 1]$.

$$H_k^T R_k^{-1} H_k - \gamma^{-2} I = \begin{bmatrix} R_k^{-1} - \gamma^{-2} & R_k^{-1} \\ R_k^{-1} & R_k^{-1} - \gamma^{-2} \end{bmatrix} \geq 0 \quad (19)$$

If else, (19) will exhibit negative definite matrix and therefore causing unreliable estimation to H_∞ filter. ∎

Even though Lemma 1 illustrates the results of a monobot, these result can reasonably aid the analysis for more complex system of 2D and 3D systems as (11), (12) act as the main algorithm for H_∞ filter. We proposed some other conditions for H_∞ filter in SLAM in the following theorem.

Theorem 1: Assume that Assumptions 1∼2 are satisfied. For $\gamma > 0$, the map uncertainties are gradually decrease if the following conditions are satisfied.

1) Equation (14) is also a PsD if the the measurement covariance noise, R is bigger than the state covariance matrix, P
2) γ^{-2} must be less than (14)
3) $H_k^T R_k^{-1} H_k - \gamma^{-2} I \geq 0$
4) *Lemma 1* is satisfied

If else, the state covariance matrix is not decreasing.

Proof: We begin the proof by define the initial state covariance matrix, $P_0 \geq 0$, the process noise, $Q_k \geq 0$ and the measurement noise, $R_k \geq 0$. To ensure the state covariance matrix converge, there are some conditions to be satisfied. First, for $\gamma > 0$, (14) is also a PsD if the measurement covariance noise, R is bigger than the state covariance matrix, P. Second, in order to realize (13), γ^{-2} must be less than

(14). Next, it is understood that, if previous condition is satisfied, then $H_k^T R_k^{-1} H_k - \gamma^{-2} I \geq 0$. Finally for $\gamma^2 > R$, (12) result in PsD matrix. These four conditions must be fulfilled to achieve convergence of the state covariance matrix. If those conditions are satisfied, then from (10), the state covariance matrix P, can be simplified as following. Let $W_k = H_k^T R_k^{-1} H_k - \gamma^{-2} I \geq 0$.

$$\begin{aligned} P_{k+1} &= [P_k^{-1} + W_k]^{-1} \geq 0 & (20) \\ P_{k+2} &= [P_{k+1}^{-1} + W_{k+1}]^{-1} & (21) \\ &= [[P_k^{-1} + W_k]^{-1} + W_{k+1}]^{-1} & (22) \\ &\leq P_{k+1} & (23) \end{aligned}$$

From the PsD properties, any submatrix of a PsD is also a PsD. Hence, the submatrix of the landmarks components also have the same characteristics.

$$P_{k+1_{mm}} \leq P_{k_{mm}} \quad (24)$$

We also found that for a case of the observation noise, $R >> \gamma$, the state covariance matrix will not be a positive definite matrix and therefore, may result in unstable estimations. ■

It is also obvious that for a case of stationary landmarks, there is no process noise incorporated in the landmark's state's estimation. Thus, all the landmark covariance is expected to be constant through the observations. In other words, it is expected theoretically in the limit, the landmark covariance yield

$$P_{k+1_{mm}} \approx P_{k_{mm}} \quad (25)$$

Unfortunately, we show that this is not actually describes for the whole state covariance matrix in the next theorem.

State covariance matrix, P is generally a representation of uncertainties for each state estimation. [3] proposed some convergence properties for Kalman filter-Based SLAM. The results are then analyzed further in the nonlinear system by [16]. For H_∞ filter in linear case SLAM, the convergence properties of a stationary robot observing landmarks are still unknown.

Theorem 2: Suppose that Theorem 1 is satisfied. For a stationary robot observing a stationary landmark m, with $\gamma > 0$, as more n-times($n > 0$) observation is made, in the limit, the whole covariance matrix is converging to

$$P_{k+1}^n = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \quad (26)$$

where

$$\begin{aligned} P_{11} &= [P_{vv}^{-1} + n(R^{-1} - \gamma^{-2} I) - nR^{-1}(R^{-1} - \gamma^{-2} I)^{-1} R^{-1}]^{-1} \\ P_{12} &= -P_{11} R^{-1} (R^{-1} - \gamma^{-2} I)^{-1} \\ P_{21} &= -(R^{-1} - \gamma^{-2} I)^{-1} R^{-1} P_{11} \\ P_{22} &= (R^{-1} - \gamma^{-2} I)^{-1} \\ &\quad + (R^{-1} - \gamma^{-2} I)^{-1} R^{-1} P_{11} R^{-1} (R^{-1} - \gamma^{-2} I)^{-1} \end{aligned}$$

If P_{11} exhibit a PsD, then the whole state covariance is decreasing. Else, the estimation is faulty.

Proof: Again we consider a 2D robot with initial covariance matrix P_0, given by the following,

$$P_0 = \begin{bmatrix} P_{vv} & 0 \\ 0 & P_{mm} \end{bmatrix} \quad (27)$$

Assume that the robot is observing one landmark m at a certain point. From (18), when the stationary robot is observing m landmarks n times, we obtained the following equations.

$$\begin{aligned} P_{k+1}^{-1} &= P_0^{-1} + n(H_k^T R_k^{-1} H_k - \gamma^{-2} I) & (28) \\ &= P_0^{-1} + n \begin{bmatrix} R^{-1} - \gamma^{-2} I & R^{-1} \\ R^{-1} & R^{-1} - \gamma^{-2} I \end{bmatrix} & (29) \end{aligned}$$

Assume that the initial state covariance matrix for the landmarks is very big. Then above equation yields

$$P_{k+1}^{-1} = \begin{bmatrix} P_{vv}^{-1} & 0 \\ 0 & P_{mm}^{-1} \end{bmatrix} + n \begin{bmatrix} R^{-1} - \gamma^{-2} I & R^{-1} \\ R^{-1} & R^{-1} - \gamma^{-2} I \end{bmatrix} \quad (30)$$

Finding the inverse matrix of (29) using the Matrix Inversion Lemma, yields

$$P_{k+1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \quad (31)$$

where

$$\begin{aligned} P_{11} &= [P_{vv}^{-1} + n(R^{-1} - \gamma^{-2} I) - nR^{-1}(R^{-1} - \gamma^{-2} I)^{-1} R^{-1}]^{-1} \\ P_{12} &= -P_{11} R^{-1} (R^{-1} - \gamma^{-2} I)^{-1} \\ P_{21} &= -(R^{-1} - \gamma^{-2} I)^{-1} R^{-1} P_{11} \\ P_{22} &= (R^{-1} - \gamma^{-2} I)^{-1} \\ &\quad + (R^{-1} - \gamma^{-2} I)^{-1} R^{-1} P_{11} R^{-1} (R^{-1} - \gamma^{-2} I)^{-1} \end{aligned}$$

As long as $R^{-1} - \gamma^{-2} I \geq 0$, (29) is a PsD. Furthermore,

$$\begin{aligned} R^{-1} - \gamma^{-2} I &\geq R^{-1}(R^{-1} - \gamma^{-2} I)^{-1} R^{-1} \\ R(R^{-1} - \gamma^{-2} I) R &\geq (R^{-1} - \gamma^{-2} I)^{-1} \end{aligned}$$

Above equation can be verified under the properties of PsD matrix. Furthermore, from (29) and Lemma 1, it can be notice that for a case of the observation noise $R >> \gamma^2$, the state covariance matrix may have a negative definite matrix that is an unexpected behavior in probabilistic SLAM. The designer must choose an appropriate value to satisfy this condition. ■

For the Kalman filter case, the state covariance is given by

$$\begin{aligned} P_{k+1}^{-1} &= P_0^{-1} + n(H_k^T R_k^{-1} H_k) & (32) \\ &= P_0^{-1} + n \begin{bmatrix} R^{-1} & R^{-1} \\ R^{-1} & R^{-1} \end{bmatrix} & (33) \end{aligned}$$

Inverting the above matrix yield

$$P_{k+1}^n = \begin{bmatrix} P_{vv} & -Pvv \\ -Pvv & R + P_{vv} \end{bmatrix} \quad (34)$$

Observing the fact obtained by Theorem 1, it implicitly determines that the state covariance matrix for H_∞ filter is slightly bigger than Kalman's filter. The second and third variables on the right hand of (28) show explicitly the increment of the state covariance matrix of H_∞. The conditions shown

TABLE I

EXPERIMENTAL PARAMETERS

γ	9
Process noise	0.000001
Observation noise,R	$\begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix}$
Random noise observation,R	$\begin{bmatrix} R_{\theta_{max}} = 0.05 \\ R_{\theta_{min}} = -0.05 \\ R_{distance_{max}} = 0.2 \\ R_{distance_{min}} = -0.2 \end{bmatrix}$
Initial Covariance(Case 1)Pvv, Pmm	0.00001, 10000
Initial Covariance(Case 2)Pvv, Pmm	0.0001, 0.001

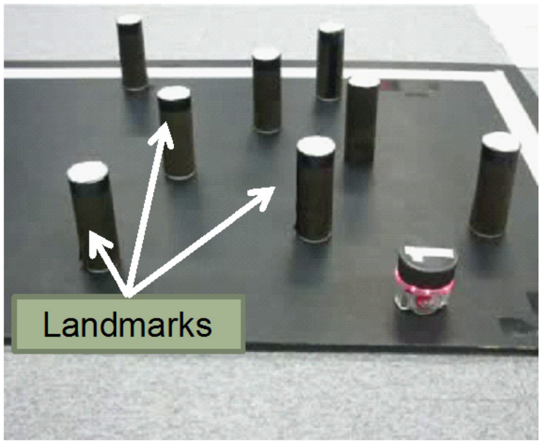

Fig. 2. Indoor experimental environment

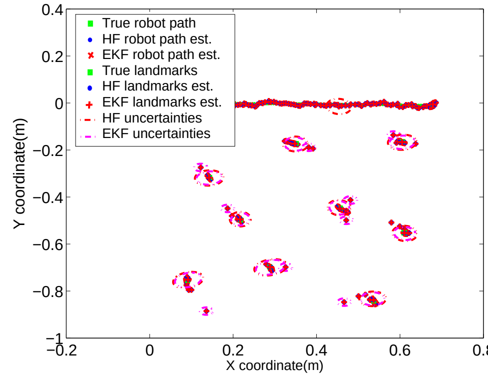

Fig. 3. Constructed map under zero mean Gaussian noises

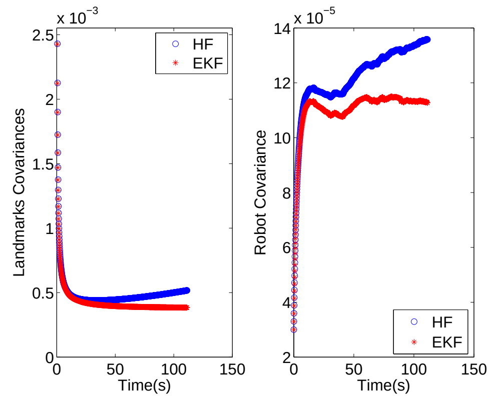

Fig. 4. Robot and landmarks covariances under zero mean Gaussian noises

in Theorem 1 clarify further that designer must choose an appropriate noise energy level and γ to achieve desired performance. Furthermore, $R^{-1} - \gamma^{-2}I$ formulate how actually γ attempt to reduce the noise effect to the system. Besides, it proves that in the limit, the landmark state covariance is bigger than the initial condition.

V. EXPERIMENTAL RESULTS

This section evaluates the proposed theorems in an indoor environment. The environment of the experiments is shown in Fig.2 which consists of an E-puck robot with some available landmarks. Two kinds of initial state covariance are used to investigate both EKF and HF filter convergence and behavior in SLAM problem. E-puck robot moves through the environment and observing its surrounding. We use a camera sensor as a virtual sensor to evaluate the performance between H_∞ filter and Kalman filter. The results should be consistent with the proposed theorems. We made some assumptions as stated below for the experiment to ensure that the characteristics and consistency are inherent as shown in the convergence theorems.

Assumption 4: Robot is in planar world

Assumption 5: Landmarks are stationary and consists of point landmarks

For the zero mean gaussian case, there are no big differences between EKF and H_∞(HF) estimations. Both filters can fairly estimate the robot path and landmarks location as seen on Fig.3. Both estimations show familiar results and capable of build the map. Looking at the state covariance matrix, Fig.4 shows that in both robot and landmarks covariance, HF state covariance matrix is converging although it showed a slightly higher covariances than Kalman filter. In this case, KF performance is better than HF.

Both Fig.5 and Fig.6 demonstrates the results of applying case 2 initial covariance into the system where the robot initial uncertainties become bigger. Both filters shows diverse estimations. But it is observable HF perform better in this case. KF estimation is erroneous even though the landmark initial covariance is small. This result proves that HF is more robust than KF for robot with bigger uncertainties. Fig.8 illustrate that HF covariances are also converging which satisfies Theorem 2 although it is bigger than KF. This is still acceptable as the estimation is better than KF.

There is also a need to investigate the case that has been proposed in Lemma 1 and Theorem 1. In the case of $\gamma^2 << R$, HF incapable to achieve better results than Kalman filter. The estimation of the robot position as shown in Fig.8 demonstrates that HF estimations exhibit erroneous results and inherently causing unreliable estimations of both robot and landmarks location. Therefore, in HF, $\gamma^2 << R$ must be satisfied to achieve desired results and performance.

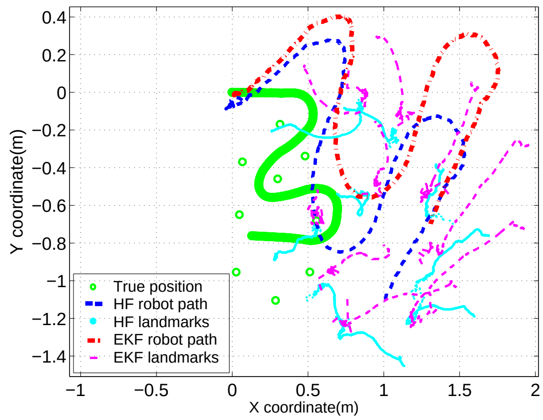

Fig. 5. Constructed map under bigger initial uncertainties

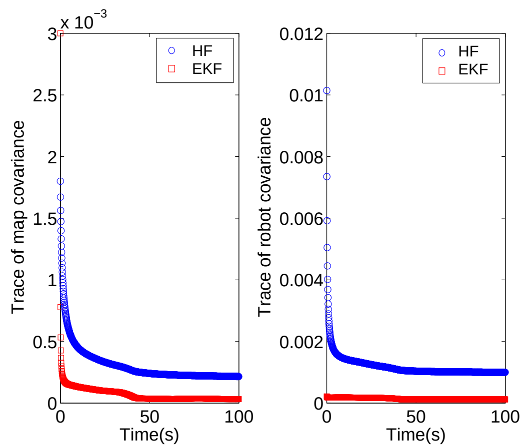

Fig. 6. Robot and landmarks covariances under bigger initial uncertainties

VI. Conclusions

H_∞ filter is still new and may need further improvement and development to achieve stable and motivating results. Even so, H_∞ filter is capable to approximate linear and non-linear system that has wide coverage and variety of noise and proven to be useful for SLAM problem. We demonstrated in this paper that the HF estimation results in better performance than KF in a case where the robot initial uncertainties is big even if the landmarks initial covariance is small. The results also consistent with the fundamental lies in H_∞ filter where the designer should consider appropriate level of weighting noise of Q, and R to achieve certain level of performance.

A. Acknowledgements

The authors gratefully acknowledge Kementerian Pengajian Tinggi Malaysia(KPT), the University Malaysia Pahang(UMP) and Mr.Tatsuya Ohtani for their continuous support and comments.

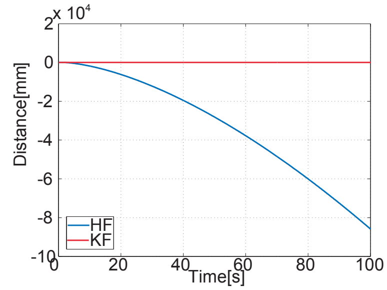

Fig. 7. The landmarks estimation when robot is stationary:Effect of $R >> \gamma$

References

[1] R. C. Smith and P. Cheeseman, On the Representation and Estimation of Spatial Uncertainty, Technical Report TR 4760 and 7239, SRI, 1985.

[2] S.Thrun, W.Burgard, D.Fox, *Probabilistic Robotics*, MIT Press, 2005.

[3] G.Dissayanake, P.Newman, S.Clark, H.Durrant-Whyte, M.Csorba, A Solution to the simultaneous localization and map building(SLAM) problem, *IEEE Transaction of Robot and Automation*, Vol.17, No.3, 2001, pp.229-241.

[4] S. Thrun, W. Burgard, , and D. Fox, "A real-time algorithm for mobile robot mapping with applications to multi-robot and 3d mapping", *In Proc. IEEE Intl. Conf. on Robotics and Automation*, Vol.1, 2000, pp. 321328.

[5] D. Hahnel, W. Burgard, and S. Thrun, "Learning compact 3d models of indoor and outdoor environments with a mobile robot", *In Proc. European workshop on advanced mobile robots*, 2001.

[6] S. Thrun, Robotic mapping: A survey, in Exploring Artificial Intelligence in the New Millenium, G. Lakemeyer and B. Nebel, Eds.Morgan Kaufmann, , 2003, pp. 135.

[7] Dan Simon, *Optimal State Estimation*, Wiley-Interscience, 2006, pp.333-367.

[8] T.Vidal-Calleja, J.Andrade-Cetto,A.Sanfeliu, "Conditions for Suboptimal filter Stability in SLAM", *Proceedings of 2004 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Sept 28-Oct.2, Sendai, Japan, 2004.

[9] D. Simon: From Here to Infinity, *Embedded Systems Programming*, Vol. 14, No. 11, 2001, pp. 20-32.

[10] A.Sabater, F.Thomas, "Set Membership Approach to the Propagation of Uncertain Geometric Information", *Proceedings of 1991 IEEE International Conference on Robotics and Automation*, 1991.

[11] A.Garulli, A.Vicino, "Set Membership Localization of Mobile Robots via Angle Measurements", *IEEE Transaction on Robotics and Automation*, Vol.17 No.4, 2001.

[12] L.M.Paz, J. Neira, "Optimal local map size for EKF-based SLAM", *IEEE/RSJ International Conference of Intelligent Robots and Systems*, pp.5019 - 5025, 2006.

[13] S.Rongchuan, M.Shugen, L Bin, W.Yuechao, "Improving consistency of EKF-based SLAM algorithms by using accurate linear approximation", *IEEE/ASME International Conference of Advanced Intelligent Mechatronics*, 2008, pp. 619 - 624.

[14] R.Martinez-Cantin, J.A Castellanos, "Bounding uncertainty in EKF-SLAM: the robocentric local approach", *IEEE Proceedings of International Conference of Robotics and Automation*, 2006, pp.430 - 435.

[15] M.E. West and V.L. Syrmos: Navigation of an autonomous underwater vehicle (AUV) using robust SLAM, Proc. 2006 IEEE CCA, 2006, pp.1801-1806.

[16] S.Huang,G.Dissayanake: Convergence and consistency Analysis for Extended Kalman filter Based SLAM", *IEEE Transaction on Robotics*, Vol.23, No.5, 2007.