Bilateral Teleoperation of Multiple Cooperative Robots with Time-Varying Delay

Nam Duc Do, Yusuke Yamashina and Toru Namerikawa

***Abstract*— This paper deals with a passive-decomposition based control of bilateral teleoperation between a single master robot and multiple cooperative slave robots under time varying delay in the communication line. At first, we decompose the dynamics of multiple slave robots into two decoupled dynamics: the Shape-System describing dynamics of the cooperative works, and the Locked-System representing overall behavior of the multiple slave robots. Second, we propose a PD control method for bilateral teleoperation to guarantee the asymptotical stability of the system for time varying delay. Finally, experimental results show the effectiveness of our proposed teleoperation.**

I. INTRODUCTION

Teleoperation systems allow person to extend their sense and manipulation capabilities to remote place. In general, slave robot is controlled to do some real tasks at the remote place by the controlled signals that send from the master side. It is composed communication channels to connect the robots and the remote environment. In bilateral control, contact information will feed back to the master side when the salve robot interacts with the remote environment, therefore the manipulation capability can be improved [1]. One absolutely unsolved problem of the control of teleoperation system is time delay in communication line. The delay may destabilize and deteriorate the transparency of the teleoperation system. In addition, the master and the slave are couple via a communication network (e.g internet), the time delay is incurred in the transmission of data between the master and the slave side. Therefore, it is necessary to design a control law to guarantee the stability of the system under communication delays. The time delay is not only constant but also variable.

Up to now, many successful control schemes have been proposed for the teleoperation system with singer master singer slave (SMSS). However, the teleoperation systems with multirobot are relative rare. In [2], [3], [4], [5] some control methods were proposed for the system with multiple mater multiple slave (MMMS). In this system, one human can control one slave robot to perform separate operation in a cooperative task, thus the system may demand a large of number of human operators if the task requires many slave robots. In [6], [7], [8], [9] the singer master multiple slave (SMMS) systems were considered, but the control methods were only proposed for the motion coordination. Both systems (MMMS and SMMS) are applied for the tasks which need the cooperation of many slave robots, such as lifting heavy objects, assembly works etc.

In the SMMS systems, there is one master robot and there are two or more slave robots. One human has to operate all slave robots at the same time by using only one master robot in a cooperative task. The control scheme for this system is not easy, especially, in the case of the movement and the contact force of each slave robot are variety. The control algorithm of only one master robot is required corresponding with the number of the slave robots. To solve above difficulties, the method based Passive-Decomposition is proposed as a technique for making two or more slave robots cooperate in the SMMS system [4]. In this work, utilizing Passive-Decomposition, the dynamics of the multiple cooperative slave robots is decomposed into decouple systems while enforcing passivity. There are two concepts: the Shape-System instructs the dynamics of the cooperative work; the Locked- System abstracts the overall dynamics of the multi-slave robots. To passive the master-slave communication delay, the scattering-based communication is utilized [10]. However in this work, neither alignment error between each slave robot position nor force reflection of them are guaranteed. On the other hand, in [11], [12] the PD control was used without the scattering conversion and the controller gains depend on the maximum round-trip delay, however, the stability is guaranteed with the communication delay.

In this paper, one control law that based on the technique of [10], [11] for the SMMS system is proposed with time varying delay in the communication line. This proposed control is also guaranteed the asymptotical stability. In [10], scattering conversion uses the PD control law with constant time delay of communication lines, however without using it for the time varying communication delay, we also achieve the stability. In our proposed control law, we assume using an individual gain for a different structure of the master and the slaves. In the independent design, a scaling power can be set to both sides of teleoperation. In addition, the teleoperation achieves an asymptotical stable with independent of time varying communication delay, the master and slave spacing errors achieve zero, the static reflection force is transferred when the cooperative slaves contact with the remote environment in this control law. In the experiment, two slave robots hold and carry one object to one desired position, and this experiment results show the effectiveness of our proposed control technique.

Nam. D. D. and Y. Yamashina are with Division of Electrical Engineering and Computer Science. Graduate School of Natural Science and Technology, Kanazawa University, Kakuma-machi Kanazawa-City, Ishikawa, Japan, 920-1192, JAPAN. namdd@scl.ec.t.kanazawa-u.ac.jp

T. Namerikawa is with the Department of System Design Engineering Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522 JAPAN. namerikawa@sd.keio.ac.jp

II. Problem Formulations

1) Dynamics of Teleoperation System: In this section, the dynamics of the SMMS system that composed one master and N slave robots can be shown by a motion equation of a general robot arm. The dynamic of the master with m-DOF and the dynamics of the i slave with n_i-DOF are shown as follows:

$$\begin{cases} M_m(q_m)\ddot{q}_m + C_m(q_m,\dot{q}_m)\dot{q}_m = \tau_m + J_m^T(q_m)F_{op} \\ M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i = \tau_i + J_i^T(q_i)F_i \end{cases} \quad (1)$$

where the subscript "m" denotes the master and the subscript "i" denotes order indexes of the slave, $q_m \in R^{m\times 1}$, $q_i \in R^{n_i\times 1}$ are the joint angle vectors, $\tau_m \in R^{m\times 1}$, $\tau_i \in R^{n_i\times 1}$ are the input torque vectors, $F_{op} \in R^{m\times 1}$ is the operational force vector, $F_i \in R^{n_i\times 1}$ are the grasping force vectors, $M_m \in R^{m\times m}$, $M_i \in R^{n_i\times n_i}$ are the symmetric and positive definite inertia matrices, $C_m(q_m,\dot{q}_m)\dot{q}_m \in R^m$, $C_i(q_i,\dot{q}_i)\dot{q}_i \in R^{n_i}$ are the centripetal and Coriolis torque vectors, $J_m(q_m) \in R^{m\times m}$, $J_i(q_i) \in R^{n_i\times n_i}$ are Jacobian matrices. However, degree of freedom of the slave is assumed to be larger than the degree of freedom of the master ($n_i \geq m$). The Jacobian matrices satisfy below assumption:

Assumption 1: The J_m and J_i are nonsingular matrices at all times in operation.

In this paper, we propose a control law for different structural teleoperation. This control law of the system may be not possible with some parameters in joint space, therefore it is useful to rewrite the master and slave robot dynamics directly in the task space. The end-effector velocities $\dot{x}_m \in R^{m\times 1}$ and $\dot{x}_i \in R^{n_i\times 1}$ in task space relate to the joint velocity $\dot{q}_m$, $\dot{q}_i$ as follows:

$$\dot{x}_k(t) = J_k(q_k)\dot{q}_k(t),\ k = m,\ i. \quad (2)$$

by father differentiation of (2) as:

$$\ddot{x}_k(t) = J_k(q_k)\ddot{q}_k(t) + \dot{J}_k(q_k)\dot{q}_k^2(t),\ k = m,\ i. \quad (3)$$

where $\ddot{x}_m \in R^{m\times 1}$ and $\ddot{x}_i \in R^{n_i\times 1}$ are the end-effector acceleration vectors. Substituting (2) and 3 into (1), we can receive the master and multiple slave robots dynamics in the task space as follows:

$$\widetilde{M}_m(q_m)\ddot{x}_m + \widetilde{C}_m(q_m,\dot{q}_m)\dot{x}_m = J_m^{-T}\tau_m + F_{op} \quad (4)$$

$$\widetilde{M}_i(q_i)\ddot{x}_i + \widetilde{C}_i(q_i,\dot{q}_i)\dot{x}_i = J_i^{-T}\tau_i + F_i \quad (5)$$

where: $\widetilde{M}_k = J^{-T}M_kJ_k^{-1}$, $\widetilde{C}_k = J_k^{-T}\{C_k - M_kJ_k^{-1}\dot{J}_k\}J_k^{-1}$, ($k = m,\ i$), x_i is end-effector of each slave robot in Cartesian coordinate system of multiple slaves. Let us denote the total degree of freedom of the N slave robots by: $n = \sum_i^N n_i$, hence the group dynamics of N slave robots can be rewritten as follows:

$$\widetilde{M}(q)\ddot{x} + \widetilde{C}(q,\dot{q})\dot{x} = \tau + F \quad (6)$$

where $x = [x_1^T,\ldots,x_N^T]^T \in R^n$, $\tau = [\tau_1^TJ_1^{-T},\ldots,\tau_N^TJ_N^{-T}]^T \in R^n$, $F = [F_1^T,\ldots,F_N^T]^T \in R^n$, and $\widetilde{M}(q) = diag[\widetilde{M}_1(q_1),\ldots,\widetilde{M}_N(q_N)] \in R^{n\times n}$, $\widetilde{C}(q,\dot{q}) = diag[\widetilde{C}_1(q_1,\dot{q}_1),\ldots,\widetilde{C}_N(q_N,\dot{q}_N)] \in R^{n\times n}$ are the inertia matrices and Coriolis matrices, respectively. It is well known that the dynamics (4) and (5) have several fundamental properties under the assumption 1 as follows:

Property 1: The inertia matrices $\widetilde{M}_k(q_k)$ ($k = m,\ i$) are symmetric and positive definite and there exists some positive constant m_{k1},m_{k2}, c_k in [13] such as:

$$0 < m_{k1} \leq \| \widetilde{M}_k \| \leq m_{k2};\ \ \| \widetilde{C}_k \| \leq c_k \| \dot{x}_k \| \quad (7)$$

Property 2: Consider an appropriate definition of the matrices $\widetilde{C}_k(q_k,\dot{q}_k)$, the matrices $\widetilde{N}_k = \dot{\widetilde{M}}_k(q_k - 2\widetilde{C}_k(q_k,\dot{q}_k))$ are skew symmetric as in [13] such that:

$$z^T\widetilde{N}_kz = 0\ (k = m,\ i) \quad (8)$$

where $z \in R^{n\times 1}$ is any vector.

Property 3: $\dot{x}_k,\ddot{x}_k$ ($k = m,i$) are bounded and $\dot{\widetilde{M}}_k,\dot{\widetilde{C}}_k$ are also bounded [14]

Communication delay is assumed as follows:

Assumption 2: Both time varying delay $T_m(t)$ and $T_s(t)$ are continuously differentiable function and possibly bounded as:

$$0 \leq T_h(t) \leq T_h^+ < \infty,\ |\dot{T}_h(t)| < \dot{T}_h^+,\ h = m,s \quad (9)$$

where $T_h^+,\dot{T}_h^+ \in R$ are upper bounds of the communication delays. Moreover, the upper bound of the round trip communication delay $T_{ms}^+ = T_m^+ + T_s^+$ is known preliminarily.

Assumption 3: The delays among all slave robots can be disregarded to be very small.

A. Control Objectives

In this paper, the SMMS system is shown in Fig. 1 with one master and two slave robots. The slave robots grasp one object to transport to a specified place according to the instruction values of a control law in the task space.

Control Objective 1: (Autonomously Grasping by Multiple Slave Robots) In this work, the grasping achievement with its definition:"relative position of the end-effectors of the slave robots becomes a certain specified shape" is achieved as the following condition:

$$x_S = x_S^d \quad (10)$$

where $x_S \in R^{n-m}$ is the relative position of the end-effector of the slaves, $x_S^d \in R^{n-m}$ is a desired position of x_S.

Control Objective 2: (Movement of Grasped Object) When the grasping is achieved, the center position between the end-effector of the slave robots is same the center position of the grasped object, then the movement of the grasped object is achieved as:

$$x_L = x_m \quad (11)$$

where $x_L = \alpha x_{L0} - C$, $x_{L0} \in R^m$ and x_m are the center position of the end-effectors and the grasped object, respectively; $\alpha \in R$ is the position scale, $C \in R^m$ is shown a translation value.

Control Objective 3: (Static Force Reflection) The teleoperation with static Force Reflection is achieved as $\dot{x}_j = \ddot{x}_j = 0 (j = m,L)$ such that:

$$F_{op} = -\beta F_L \quad (12)$$

where F_L is the contact force of cooperative-slave, $\beta > 0 \in R$ is a positive scalar and it expresses a force scaling effect.

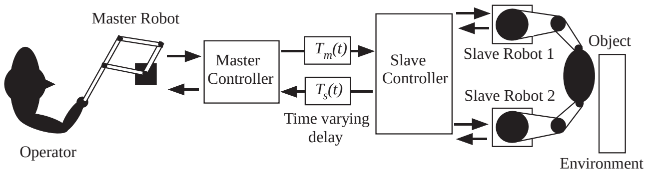


Fig. 1. SMMS Teleoperation System

III. CONTROL SYSTEM DESIGN

In this section, to achieve above Control Objectives, we propose a control law for the SMMS system.

A. Passive-Decomposition

First, base on Passive-Decomposition that was introduced by D. Lee [10], the dynamic of multiple slave robots is decomposed into two decouple systems: the Shape-System describing "movement of the multiple slaves with grasping object" and the Locked-System describing "movement of the multiple slaves according to the instruction from the master". Utilizing the Passive-Decomposition, the velocity of multiple slave robots is rewritten with each system as follows:

$$\dot{x} = S^{-1}\begin{bmatrix}\dot{x}_S\\ \dot{x}_L\end{bmatrix} \tag{13}$$

where $\dot{x}_S \in R^{n-m}$ and $\dot{x}_L \in R^m$ are velocities of the Shape-System and the Locked-System, respectively. S is the non-singular decomposition matrix. The matrix S is also a positive matrix of a decoupling shape and locked system. In the following formula of $S^{-T}\widetilde{M}S^{-1}$, the non-diagonal terms become 0 as:

$$S^{-T}\widetilde{M}S^{-1} = \begin{bmatrix}M_S & 0\\ 0 & M_L\end{bmatrix} \tag{14}$$

where $M_S \in R^{(n-m)\times(n-m)}$, $M_L \in R^{m\times m}$ are inertia matrices of the Shape-System and the Locked-System, respectively. In the fact that, $\dot{x}_s$ and $\dot{x}_L$ are defined for satisfying (14). In addition, a local compensation of impedance shaping is necessary. The reflection forces from environment relate with the control input of slave dynamics of the Shape-System and the Locked-System as follows:

$$\begin{bmatrix}F_S\\ F_L\end{bmatrix} = S^{-T}\begin{bmatrix}F_1\\ F_2\end{bmatrix},\ \begin{bmatrix}\tau_S\\ \tau_L\end{bmatrix} = S^{-T}\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} \tag{15}$$

from above definitions, we define:

$$\begin{bmatrix}C_S & C_{SL}\\ C_{LS} & C_L\end{bmatrix} = S^{-T}\widetilde{M}\frac{d}{dt}(S^{-1}) + S^{-T}\widetilde{C}S^{-1} \tag{16}$$

note (6), the Passive-Decomposition form is written as:

$$M_S(q)\ddot{x}_S + C_S(q,\dot{q})\dot{x}_S + C_{SL}(q,\dot{q})\dot{x}_L = \tau_S + F_S \tag{17}$$
$$M_L(q)\ddot{x}_L + C_L(q,\dot{q})\dot{x}_L + C_{LS}(q,\dot{q})\dot{x}_S = \tau_L + F_L \tag{18}$$

where the subscript "S" denotes the Shape-System and the subscript "L" denotes the Locked-System. The above dynamic equations include friction terms $C_{SL}(q,\dot{q})\dot{x}_L$ and $C_{LS}(q,\dot{q})\dot{x}_S$, however, ignore the remote control by the human, decoupling of the Shape-System and the Locked-System is desired for the slave that maybe autonomous grasping. Therefore, the decoupling control inputs are given:

$$\tau_S = C_{SL}(q,\dot{q})\dot{x}_L + \tau_S' \tag{19}$$
$$\tau_L = C_{LS}(q,\dot{q})\dot{x}_S + \tau_L' \tag{20}$$

where τ_S', τ_L' are new control inputs. Substituting (19), (20) into (17), (18), we get:

$$M_S(q)\ddot{x}_S + C_S(q,\dot{q})\dot{x}_S = \tau_S' + F_S \tag{21}$$
$$M_L(q)\ddot{x}_L + C_L(q,\dot{q})\dot{x}_L = \tau_L' + F_L \tag{22}$$

hence, two above dynamics become a decoupling.

These dynamics are similar to the normal dynamics and some properties are given as follows:

Property 4: $M_i(q)(i = S,L)$ is a positive symmetric matrix, and there exists some constant parameters with below relationship as :

$$0 < m_{i1} \leq \| M_i \| \leq m_{i2}$$
$$\| C_i \| \leq c_i \| \dot{x}_i \| \tag{23}$$

Property 5: $\dot{M}_i(q) - 2C_i(q,\dot{q})$ $(i = S,L)$ is skew-symmetric matrix.

Property 6: $\dot{x}_i, \ddot{x}_i$ $(i = S,L)$ are bounded and $\dot{M}_i, \dot{C}_i$ are also bounded.

Proof: In Properties 4 and 6, M_i, C_i $(i = S,L)$ are defined by (14) and (16), respectively. We also can see from S, Properties 1 and 3.

From Property 5, we can get:

$$\begin{bmatrix}\dot{M}_S - 2C_S & -2C_{SL}\\ -2C_{LS} & \dot{M}_L - 2C_L\end{bmatrix}$$
$$= \frac{d}{dt}(S^{-T}\widetilde{M}S^{-1}) - 2S^{-T}\widetilde{M}\frac{d}{dt}(S^{-1}) - 2S^{-T}\widetilde{C}S^{-1} \tag{24}$$

Using above skew-symmetric property of $\dot{\widetilde{M}} - 2\widetilde{C}$ and the symmetric property of $\widetilde{M}$, we get:

$$\begin{bmatrix}\dot{M}_S - 2C_S & -2C_{SL}\\ -2C_{LS} & \dot{M}_L - 2C_L\end{bmatrix} = -\begin{bmatrix}\dot{M}_S - 2C_S & -2C_{SL}\\ -2C_{LS} & \dot{M}_L - 2C_L\end{bmatrix}^T \tag{25}$$

therefore, the skew-symmetric matrices $\dot{M}_S - 2C_S$, $\dot{M}_L - 2C_L$ are equivalence. ■

Properties 4 $\sim$ 6 denote the feature of motion equation of normal robots, otherwise, we can applied them for the control law of abundance robots.

The following assumptions are from (1), (21), (22) and used in next stability analysis section.

Assumption 4: In the control of the Locked-System that includes the environment and grasping object. Following the model of the passive system, the velocities $\dot{x}_m$, $\dot{x}_L$ are system inputs, the force F_{op}, F_L are system outputs. Moreover, the energy is limited by the function with velocity of the master in the Locked- System.

Assumption 5: The velocities $\dot{x}_m$, $\dot{x}_L$ equal zero for $t < 0$

B. Proposal Control Law

Concerning the control law of the Shape-System (21), the Control objective of this system is: $x_S = x_S^d$, then the position tracking with this control law is shown as follows:

$$\begin{aligned}\tau_S^{'} =& M_S\{\ddot{x}_S^d(t) - K_d^S(\dot{x}_S - \dot{x}_S^d(t)) - K_P^S(x_S - x_S^d(t))\} \\ &+ C_S\dot{x}_S - F_S\end{aligned} \tag{26}$$

Substituting (26) into (21) we obtain the following closed-loop systems:

$$\begin{aligned}&\ddot{e} + K_d^S\dot{e} + K_P^S e = 0, \\ &e = x_S - x_S^d\end{aligned} \tag{27}$$

where K_d^S, K_P^S are positive definite diagonal gain matrices.

Remark 1: In the control law of this work, information of grasped object is necessary for position control. Moreover, to satisfy the Control objective 1 (10), the object is assumed to be not too hard.

Considering the coupling control of the Locked-System and the master. Note the Control objective: $x_L = x_m$, the control law is defined as:

$$\tau_L^{'} = -K_d^L\dot{x}_L - K_P^L(x_L - x_m(t - T_m(t))) \tag{28}$$

$$\tau_m = J_m^T\{-K_d^m\dot{x}_m - K_P^m(x_m - x_L(t - T_s(t)))\} \tag{29}$$

Substituting above control law into the Locked-System (22) and dynamic equation of the master (4), we obtain a closed-loop system as follows:

$$\begin{aligned}&M_L(q)\ddot{x}_L + C_L(q,\dot{q})\dot{x}_L \\ &\quad= -K_d^L\dot{x}_L - K_P^L(x_L - x_m(t - T_m(t))) + F_L\end{aligned} \tag{30}$$

$$\begin{aligned}&\widetilde{M}_m(q_m)\ddot{x}_m + \widetilde{C}_m(q_m,\dot{q}_m)\dot{x}_m \\ &\quad= -K_d^m\dot{x}_m - K_P^m(x_m - x_L(t - T_s(t))) + F_{op}\end{aligned} \tag{31}$$

where K_P^j, K_d^j $(j = m, L)$ are gains and defined as follows:

$$\left\{\begin{array}{l} K_P^m = k_m K_P \\ K_P^L = k_L K_P \end{array}\right., \quad \left\{\begin{array}{l} K_d^m = k_m K_d \\ K_d^L = k_L K_d \end{array}\right. \tag{32}$$

where $K_P \in R^{n\times n}$, $K_d \in R^{n\times n}$ are positive definite diagonal control gains; $k_m > 0, k_L > 0$ are constant gains of scalar that designed separately on the master and the slave side.

IV. Stability Analysis

A. Stability of Shape-System

The below theorem consists of the Shape-System.

Theorem 1: Consider the closed-loop Shape-System (27) and Assumption 4, desired value of relative position of spaces between the slave robots is conversed as follows:

$$e = x_S - x_S^d \rightarrow 0 \quad as\ t \rightarrow \infty \tag{33}$$

Proof: The equation (27) can be rewritten as follows:

$$\begin{bmatrix}\dot{e} \\ \ddot{e}\end{bmatrix} = \phi\begin{bmatrix}e \\ \dot{e}\end{bmatrix}, \quad \phi = \begin{bmatrix}0 & I \\ -K_P^S & -K_d^S\end{bmatrix} \tag{34}$$

where K_P^S, K_d^S are positive diagonal matrices, eigenvalue ϕ becomes negative, therefore following errors of position and velocity are achieved:

$$e = x_S - x_S^d \rightarrow 0 \quad as \quad t \rightarrow \infty \tag{35}$$

$$\dot{e} = \dot{x}_S - \dot{x}_S^d \rightarrow 0 \quad as \quad t \rightarrow \infty \tag{36}$$

it means the Control Objective 1 is achieved and the autonomous grasping of multiple slaves is also achieved. ■

B. Stability of Locked-System

The following theorem consists of the dynamics (30), (31).

Theorem 2: The equations (30), (31) indicate the teleoperation system, the Assumption 1∼5 are approved and the control gains K_P, K_d are used as below:

$$K_P < \frac{2}{T_{ms}^{+}}K_d \tag{37}$$

here, the velocities of the master and the slave are asymptotical convergent to origin, the position error is unbounded, then the system becomes an asymptotical stable.

Proof: State vector $x(t) = [\dot{x}_m^T, \dot{x}_L^T, x_e^T]^T$ is used. We define a Lyapunov function for the system as:

$$\begin{aligned}V(x(t)) =& k_m^{-1}\dot{x}_m^T(t)\widetilde{M}_m\dot{x}_m(t) + k_L^{-1}\dot{x}_L^T(t)M_L\dot{x}_L(t) \\ &+ x_e^T(t)K_P x_e(t) - 2k_m^{-1}\int_0^t F_L^T(\xi)\dot{x}_L(\xi)d\xi \\ &- 2k_L^{-1}\int_0^t F_{op}^T(\xi)\dot{x}_m(\xi)d\xi\end{aligned} \tag{38}$$

where $\widetilde{M}_m, M_L, K_P$ are positive definite matrices, $k_m, k_L > 0$. Following the Assumption 4, the environment and the manipulator are passive, then $V(x(t))$ is the positive function. The derivative of above Lyapunov function along trajectories of the system (30), (31) with concerning Properties 2 and 5 as:

$$\begin{aligned}\dot{V} =& -2\dot{x}_m^T K_d\dot{x}_m + 2\dot{x}_m^T K_P(x_L(t - T_s(t)) - x_L) \\ &- 2\dot{x}_L^T K_d\dot{x}_L + 2\dot{x}_L^T K_P(x_m(t - T_m(t)) - x_m)\end{aligned} \tag{39}$$

applying Leibniz-Newton formula:

$$x_i(t - T_h(t)) - x_i = -\int_0^{T_h(t)}\dot{x}_h(t - \xi)d\xi, \quad (h = m, s) \tag{40}$$

substituting (40) in to (39), we get:

$$\begin{aligned}\dot{V} =& -2\dot{x}_m^T K_d\dot{x}_m - 2\dot{x}_m^T K_P\int_0^{T_s(t)}\dot{x}_L(t - \xi)d\xi \\ &- 2\dot{x}_L^T K_d\dot{x}_L - 2\dot{x}_L^T K_P\int_0^{T_s(t)}\dot{x}_m(t - \xi)d\xi\end{aligned} \tag{41}$$

The second term at the right side of (41) is transformed as follows:

$$-2\dot{x}_m^T K_P\int_0^{T_s(t)}\dot{x}_L(t - \xi)d\xi = -\sum_{j=1}^{n}K_{Pj}2\dot{x}_{mj}\int_0^{T_s(t)}\dot{x}_{Lj}(t - \xi)d\xi \tag{42}$$

where $\dot{x}_{mj}, \dot{x}_{Lj}, K_{Pj}$ are velocities of the master and slave (following the j axis) and positional control gains, respectively. In (42), applying Young and Schawartz inequality for the term in the right side, then note the inequality $T_s \le T_s^+$, we get:

$$\begin{aligned}-2\dot{x}_{mj}\int_0^{T_s(t)}\dot{x}_{Lj}(t - \xi)d\xi &\le T_s^+\dot{x}_{mj}^2 + \frac{1}{T_s^+}\left\{T_s\int_0^{T_s(t)}\dot{x}_{Lj}^2(t - \xi)d\xi\right\} \\ &\le T_s^+\dot{x}_{mj}^2 + \int_0^{T_s^+}\dot{x}_{Lj}^2(t - \xi)d\xi\end{aligned} \tag{43}$$

Therefore, (42) is rewritten as follows:

$$-2\dot{x}_m^T K_P \int_0^{T_s(t)} \dot{x}_L(t-\xi)d\xi \leq \sum_{j=1}^{n} K_{Pj}\Big\{T_s^+ \dot{x}_{mj}^2 + \int_0^{T_s^+} \dot{x}_{Lj}^2(t-\xi)d\xi\Big\}$$
$$= T_s^+ \dot{x}_m^T K_P \dot{x}_m + \int_0^{T_s^+} \dot{x}_L^T(t-\xi)K_P\dot{x}_L(t-\xi)d\xi \quad (44)$$

Similar to (41), the fourth term in the right side can also be rewritten. We receive below inequality from (41) as:

$$\begin{aligned}\dot{V} \leq & -2\dot{x}_m^T K_d \dot{x}_m - 2\dot{x}_L K_d \dot{x}_L \\ & + T_s^+ \dot{x}_m^T K_P \dot{x}_m + \int_0^{T_s^+} \dot{x}_L^T(t-\xi)K_P\dot{x}_L(t-\xi)d\xi \\ & + T_m^+ \dot{x}_L^T K_P \dot{x}_L + \int_0^{T_m^+} \dot{x}_m^T(t-\xi)K_P\dot{x}_m(t-\xi)d\xi \quad (45)\end{aligned}$$

here, integrating both side of above inequality $[0,\ t]$, we get:

$$\begin{aligned}\int_0^t \dot{V}d\tau \leq & -2\int_0^t \dot{x}_m^T K_d \dot{x}_m d\tau - 2\int_0^t \dot{x}_L^T K_d \dot{x}_L d\tau \\ & + \int_0^t T_s^+ \dot{x}_m^T K_P \dot{x} d\tau + \int_0^t T_m^+ \dot{x}_L^T K_P \dot{x}_L d\tau \\ & + \int_0^t \int_0^{T_s^+} \dot{x}_L^T(\tau-\xi)K_P\dot{x}_L(\tau-\xi)d\xi d\tau \\ & + \int_0^t \int_0^{T_m^+} \dot{x}_m^T(\tau-\xi)K_P\dot{x}_m(\tau-\xi)d\xi d\tau \quad (46)\end{aligned}$$

here, the fifth and sixth terms of right side in (46) can be transformed by a simple calculation as follows:

$$\begin{aligned}&\int_0^t \int_0^{T_s^+} \dot{x}_L^T(\tau-\xi)K_P\dot{x}_L(\tau-\xi)d\xi d\tau \\ &\quad \leq T_s^+ \int_0^t \dot{x}_L^T(\tau)K_P\dot{x}_L(\tau)d\tau \quad (47)\end{aligned}$$

$$\begin{aligned}&\int_0^t \int_0^{T_m^+} \dot{x}_m^T(\tau-\xi)K_P\dot{x}_m(\tau-\xi)d\xi d\tau \\ &\quad \leq T_m^+ \int_0^t \dot{x}_m^T(\tau)K_P\dot{x}_m(\tau)d\tau \quad (48)\end{aligned}$$

Substituting (47), (48) into (46), we obtain:

$$\begin{aligned}\int_0^t \dot{V}d\tau \leq & -\int_0^t \dot{x}_L^T\{2K_d - T_{ms}^+ K_P\}\dot{x}_L d\tau \\ & -\int_0^t \dot{x}_m^T\{2K_d - T_{ms}^+ K_P\}\dot{x}_m d\tau \quad (49)\end{aligned}$$

Therefore, we can choose the gain K_P, K_d to satisfy (37), thus (49) is semi-negative with denoting $\dot{x}_m, \dot{x}_L \in \mathscr{L}_2$. Moreover, applying Properties 1, 4 and the Assumption 4 for the dynamics of system (30), (31), we conclude that the signal $\ddot{x}_m, \ddot{x}_L \in \mathscr{L}_\infty$. Thus, using lemma of [14], this implies that $\lim_{t\to\infty}\dot{x}_m = \lim_{t\to\infty}\dot{x}_L = 0$, and using Properties 3, 6, we also can conclude $\dddot{x}_m, \dddot{x}_L \in \mathscr{L}_\infty$. Hence, invoking Barbalat's lemma [15], $\ddot{x}_m, \ddot{x}_L$ are uniformly continuous; $\lim_{t\to\infty}\dot{x}_m = \lim_{t\to\infty}\dot{x}_L = 0$ and $\lim_{t\to\infty}\ddot{x}_m = \lim_{t\to\infty}\ddot{x}_L = 0$. Therefore the system is asymptotic stable. ■

In addition, two below corollaries that relate above theorem as:

Corollary 1: It implies that the teleoperation system described by (4), (22) satisfy the Theorem 2. When $F_L = 0$, the master and slaves spacing error achieve to zero as below:

$$x_e = x_m - x_L \to 0 \quad as \quad t \to \infty \quad (50)$$

Proof: when $F_L = 0$, equation (30) as:

$$K_P(x_L - x_m(t - T_m(t))) = 0 \quad (51)$$

Moreover, using Leibniz-Newton formula, following equation is achieved:

$$K_P\Big\{x_e - \int_{t-T_m}^{t} \dot{x}_m dt\Big\} = 0 \quad (52)$$

where $\lim_{t\to\infty}\dot{x}_m = 0$, K_P is a positive symmetric matrix,

$$\lim_{t\to\infty} x_e = 0 \quad (53)$$

hence the position error of the master and the slave robots is to zero. Thus, the Control Objective 2 is achieved. ■

Corollary 2: It implies that the teleoperation system described by (4), (22) satisfies Theorem 2. We obtain that the scaled reflection force from remote environment is accurately transmitted to the slave robot side as follows:

$$F_{op} = -\beta F_L, \quad (\beta = \frac{k_m}{k_L}) \quad (54)$$

Proof: From Theorem 2, $\lim_{t\to\infty}\ddot{x}_m = \lim_{t\to\infty}\ddot{x}_L = \lim_{t\to\infty}\dot{x}_m = \lim_{t\to\infty}\dot{x}_L = 0$, and concerning about (30), (31) we can obtained:

$$\begin{cases} F_{op} = K_P^m(x_m - x_L) = k_m K_P(x_m - x_L) \\ F_L = K_P^L(x_L - x_m) = -k_L K_P(x_m - x_L) \end{cases} \quad (55)$$

From equation (55), we get above expression (54)

$$F_{op} = -\beta F_L, \quad (\beta = \frac{k_m}{k_L})$$

we should choose the design parameters of scalar k_m, k_L for power scaling. Therefore, the static reflection force is achieved. ■

V. EVALUATION BY CONTROL EXPERIMENTS

A. Impedance Shaping

In this paper, the SMMS system was constructed with two of 2-DOF serial-link arm of slave robots. Some parameters x_S, x_S^d, x_L are defined as follows:

$$x_S = \bar{x}_1 - \bar{x}_2 = \begin{bmatrix} x_1 - x_2 \\ y_1 - y_2 \end{bmatrix} \quad (56)$$

$$x_S^d = \begin{bmatrix} d \\ 0 \end{bmatrix} \quad (57)$$

$$x_L = \alpha\frac{\bar{x}_1 + \bar{x}_2 - C}{2} = \frac{\alpha}{2}\begin{bmatrix} x_1 + x_2 - c \\ y_1 + y_2 \end{bmatrix} \quad (58)$$

where $C = [c \quad 0]^T$, $\bar{x}_1 = [x_1 \ y_1]^T$, $\bar{x}_2 = [x_2 \ y_2]^T$; from (56) and (58) we get:

$$\begin{bmatrix} \dot{x}_S \\ \dot{x}_L \end{bmatrix} = \begin{bmatrix} \dot{\bar{x}}_1 - \dot{\bar{x}}_2 \\ \frac{\alpha}{2}(\dot{\bar{x}}_1 + \dot{\bar{x}}_2) \end{bmatrix} = \begin{bmatrix} I & -I \\ \frac{\alpha}{2}I & \frac{\alpha}{2}I \end{bmatrix}\begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \end{bmatrix} \quad (59)$$

We define the decomposition matrix S as follows:

$$S = \begin{bmatrix} I & -I \\ \frac{\alpha}{2}I & \frac{\alpha}{2}I \end{bmatrix} \quad (60)$$

However, the non-diagonal term has remained without filling (14) with the decomposition S. Thus, the linearization of the slaves with the impedance shaping is given:

$$\tau_i = J_i^T\{M_i H_0^{-1}(\tau_i' + F_i) + F_i + C_i\dot{x}_i\} \quad (i = 1, 2) \quad (61)$$

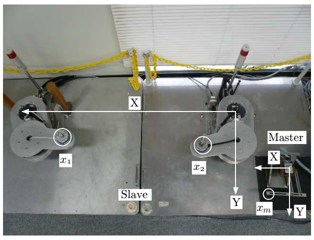


Fig. 2. Experimental setup.

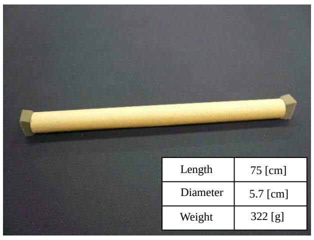


Fig. 3. Grasping object.

where $\tau_i^{'}$ is a new control input, H_0 is inertia matrix of device. To satisfy (14), by a simple calculation, we can receive the slave 1 and 2 with same inertia matrix. Therefore, substituting $M_1 = M_2 = H$ into slave dynamic (61), we obtain:

$$\begin{bmatrix} H & 0 \\ 0 & H \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} \tau_1^{'} \\ \tau_2^{'} \end{bmatrix} + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \tag{62}$$

from (14), we get:

$$\begin{aligned} S^{-T}MS^{-1} &= \begin{bmatrix} \frac{1}{2}I & -\frac{1}{2}I \\ \frac{1}{\alpha}I & \frac{1}{\alpha}I \end{bmatrix} \begin{bmatrix} H & 0 \\ 0 & H \end{bmatrix} \begin{bmatrix} \frac{1}{2}I & \frac{1}{\alpha}I \\ \frac{1}{2}I & -\frac{1}{\alpha}I \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{2}H & 0 \\ 0 & \frac{1}{\alpha^2}H \end{bmatrix} = \begin{bmatrix} M_S & 0 \\ 0 & M_L \end{bmatrix} \end{aligned} \tag{63}$$

In addition, since (14) is satisfied, it is early to see that the Shape-System and the Locked-System to be decoupling. If the Passive-Decomposition is denoted by (62), we receive:

$$\begin{bmatrix} M_S & 0 \\ 0 & M_L \end{bmatrix} \begin{bmatrix} \ddot{x}_S \\ \ddot{x}_L \end{bmatrix} = \begin{bmatrix} \tau_S^{'} \\ \tau_L^{'} \end{bmatrix} + \begin{bmatrix} F_S \\ F_L \end{bmatrix} \tag{64}$$

Therefore, by the definition of x_S, x_L mentioned above, the Shape- System and the Locked-System are decoupling by the impedance shaping only.

B. Evaluation by Control Experiments

In this section, the effectiveness of the proposal methodology is verified by the control experiment. In the experiment, the SMMS system is constructed by one master with two DOFs parallel link type arm and two slaves with two-two DOFs series link type arms. The experiment setup is shown in Fig. 2. The cylindrical grasping object is used and shown in Fig. 3. We can measure the operational force F_{op} and environment reflecting force F_L by using the force sensors. For implementation of the controllers and communication lines, we utilise a dSPACE digital control system (dSPACE Inc.). All experiments have been done with the artificial time varying communication delays and the sampling time is $1[ms]$:

$$\begin{cases} T_m(t) = 0.1\sin t + 0.14 \ [s] \\ T_s(t) = 0.05\sin t + 0.1 \ [s] \end{cases} \tag{65}$$

From above equation, maximum round-trip delay is [0.39]. To satisfy (37) the controller gains are chosen as: $K_P = diag(30,35)$, $K_d = diag(6,7)$, $k_m = 1, k_L = 10$, $K_P^S = diag(400,400)$, $K_d^S = diag(50,50)$. Two kind of experimental conditions are given as follows:

Case 1: Control the grasping object without any contact with remote environment

Case 2: Control the grasping object in contact with remote environment

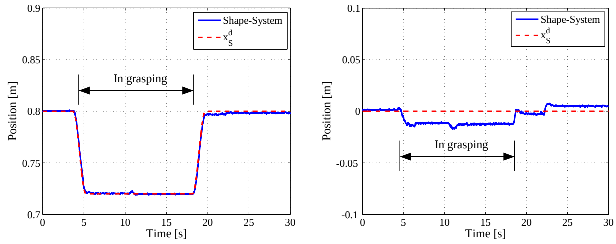
However, in actual experiments, it is difficult for entirety time synchronization on mater and slave side in the system configuration. The data that received from master and the data of slave that measured from slave side need be compared, especially the position data on the slave side. In addition, the force data is not sent and received, then the measurement value is used. Therefore, the gap of the time axis is caused for the force data to be not same at the both sides of teleoperation. Moreover, there is not sensor in the parallel link type arm of the master robot, thus the value of human force F_{op} is presumed from the input torque ($F_{op} = J_m^{-T}\tau_m$).

The experiment results of Case 1 are shown in Figs. 4-6. The Fig. 4 shows time responses of end-effector position of slave of the Shape-System, Fig. 5 shows the time responses of end-effector of the master of the Locked-System. In the Fig. 4, we can conclude that the relative position between slaves following a target trajectory with grasping object is achieved. And in the Fig. 5, we also conclude that the grasping object at the center position of slaves is able to transported following the end-effector of the master. The object is presumed to mix with closed links of slaves. When grasping, the distance between slaves is narrowed. However, this distance narrowed by each slave robot is different when the object is held deflection. The force of the Shape-System and the Locked-System in this case are shown in Fig. 6. We can see that the Fig. 6 (b) shows the force data when the object is transferred without contact with the remote environment.

The experiment results of Case 2 are shown in Figs. 8-9. The object comes and contacts with the remote environment following vertical Y axis as shown in Fig. 7. Fig. 8 shows the time responses of end-effector position of the Locked-System with the master, the Fig. 9 shows the time responses of reflection force from environment. In the Fig. 9, the grasping object comes and contacts with environment in case of the master and the slave are stationary states. Moreover, the reflecting force is transmitted in scale environment with $F_{op} = -\beta F_L$ ($\beta = 1/10$).

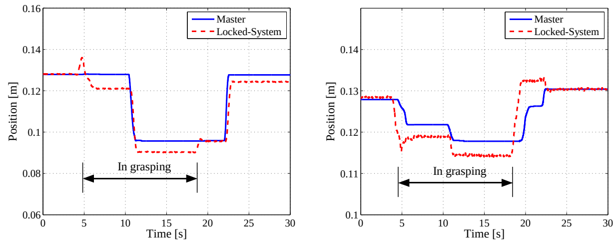
VI. CONCLUSIONS

In this paper, we proposed the control method that guarantee the asymptotical stability to the SMMS system with time varying delay in the communication lines. The proposal control law shows that the system became an asymptotically stable for the communication of time varying delay by using PD control and apply to Passive-Decomposition. This method resolves the dynamics of multiple slaves system such as the Shape-System dynamic and the Locked-System dynamic of the control law. Moreover, the proposal control law can be used to achieve an autonomous grasping object by multiple slave and the transportation of the object by the control experiment. In this work, the slaves are possible to hold even if unknown object or the width extendable



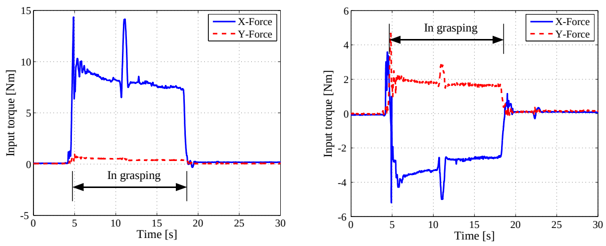
(a) X-Position (b) Y-Position

Fig. 4. Position of Shape-System and x_S^d.



(a) X-Position (b) Y-Position

Fig. 5. Position of Master and locked-System.



(a) Shape-System (b) Locked-System

Fig. 6. Force.

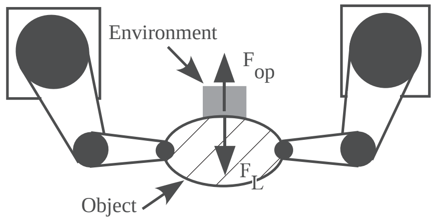
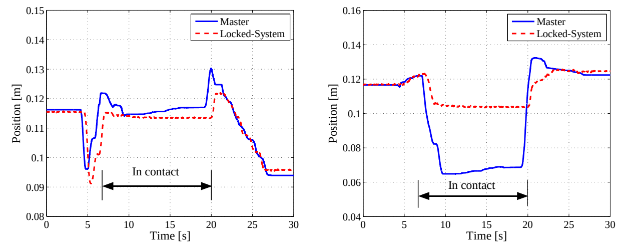
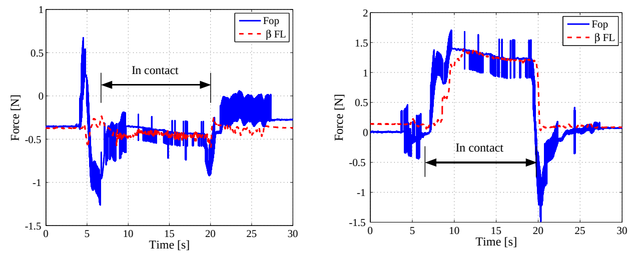


Fig. 7. Experimental Setup in Case 2



(a) X-Position (b) Y-Position

Fig. 8. Position of Master and Locked-System.



(a) X-Force (b) Y-Force

Fig. 9. Force of Operator and scale Locked-System.

of object if it can be held by the force control. The force information on the grasping object is necessary for the position control law to keep the object to be held.

Finally, several experiment results show the effectiveness of our proposal control method.

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