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Quantum beat and entanglement of multi-qubits interacting with a common reservoir

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Abstract

The qubits can be entangled when they interact with a common Ohmic reservoir. We analyze how the reservoir-induced entanglement of qubits can be observed as the beat signal in the decay curve of the macroscopic polarization. The origin of this effect is the Lamb phase shift on the qubit array. We quantify the amount of the reservoir-induced entanglement and show how to experimentally evaluate it from the decay curve of the macroscopic polarization. We discuss how the beat signal can be discriminated from the other kinds of beat signals. We also show that our analysis can be used to estimate the reservoir characteristics.

Keywords: Decoherence; Free induction decay; Quantum beat; Entanglement

1. Introduction

The decoherence of two-state system (qubit) has recently been major concern in the context of quantum information processing. A commonly used model is the spin-boson Hamiltonian which yields a good approximation in most cases. In most of previous investigations, the qubits are assumed to couple independently to separate reservoirs. It is, however, often the case where each qubit interacts with a common reservoir resulting in the cooperative decoherence [1–5]. Such a decoherence was extensively studied in the case of two-qubit in Ref. [3], clarifying that there exist the coherence-preserving states and the super-decoherence states. In Refs. [4,5], it was shown that the qubits can be entangled by the interaction via a common reservoir even if they do not interact directly with each other, shedding new light on the creation mechanism of entanglement for quantum information processing purposes. While Ref. [5] deals with a single-mode thermal reservoir, Ref. [4] assumes a thermal reservoir with many modes. The fact that the qubit can be entangled even in the latter case is rather

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surprising since interactions with infinitely many modes lead typically to very rapid decoherence, and hence to a classical mixed state without any quantum entanglement. In Ref. [4], a model is also proposed to observe this effect. The model consists of two double-well quantum dots (QDs) enclosed in a cavity, where two QDs interact only with the TM modes of the cavity. It is, however, not clear how to measure the entanglement quantitatively in practical experimental settings. In the other previous studies [1–3], the detail of dynamics and the connection to practical experiments have not been clarified either.

In this paper, we study the dynamics of a multi-qubit system interacting with a common reservoir characterized by the Ohmic spectrum of infinitely many modes. We show how one can experimentally quantify the reservoir-induced entanglement between qubits by observing the decay of the macroscopic polarization which is accessible by standard techniques such as the four-wave mixing spectroscopy for excitons, NMR for spins, and so on. In contrast to the case where the qubits couple independently to separate reservoirs, the decay of the macroscopic polarization exhibits oscillations, i.e. a beat signal reflecting the entanglement between qubits. We clarify the relation between this beat signal and the amount of the reservoir-induced entanglement in the two-qubit case. The mechanism causing the beat signal can be explained in terms of the Lamb phase shift, which was originally found in atomic physics. We discuss how to discriminate this beat signal from other kinds of beat signals encountered commonly in solid state systems. We also demonstrate that our analysis will be useful to estimate the reservoir characteristics such as reservoir boson spectrum and coherent length (time) of reservoir bosons.

2. Calculation

We assume that the following interaction pictured spin-boson Hamiltonian of L qubits

$$\tilde{H}_{\rm QR}^{(L)}(t) = \hbar \sum_{j=1}^{L} \hat{\sigma}_z^{[j]} \sum_{\boldsymbol{p}} (g_{\boldsymbol{p}}^{[j]}(t) \hat{B}_{\boldsymbol{p}}^{\dagger} + \text{h.c.}), \tag{1}$$

where $g_{\boldsymbol{p}}^{[j]}(t) \equiv g_{\boldsymbol{p}} {\rm e}^{-{\rm i}\boldsymbol{p}\cdot\boldsymbol{r}_j + {\rm i}\Omega_{\boldsymbol{p}} t}$ is the coupling function between the jth qubit at location $\boldsymbol{r}_j$ described by the Pauli spin-z operator $\hat{\sigma}_z^{[j]}$ and the reservoir boson with the momentum $\boldsymbol{p}$ and the angular frequency $\Omega_{\boldsymbol{p}}$ described by the creation operator $\hat{B}_{\boldsymbol{p}}^{\dagger}$. We assume that the qubits are initially in the separable tensor product state the ground state $|0\rangle$, and the reservoir is in the thermal state, being separated from the qubits,

$$\hat{\rho}_{\rm R} = \sum_{\boldsymbol{p}} \left(1 - \exp\left[-\frac{\hbar\Omega_{\boldsymbol{p}}}{k_{\rm B}T}\right]\right) \exp\left[-\frac{\hat{n}_{\boldsymbol{p}}\hbar\Omega_{\boldsymbol{p}}}{k_{\rm B}T}\right], \tag{2}$$

where $k_{\rm B}$ and T are Boltzmann constant and temperature, respectively, and $\hat{n}_{\boldsymbol{p}} = \hat{B}_{\boldsymbol{p}}^{\dagger}\hat{B}_{\boldsymbol{p}}$. The qubits are first excited by the $\pi/2$ pulse with the wave number vector $\boldsymbol{k}$ at time $t = 0$. The qubits and the reservoir then evolve according to the unitary operator [6]

$$\tilde{U}_{\rm QR}^{(L)}(t) = \exp\left[\sum_{\boldsymbol{p}} \sum_{j=1}^{L} \hat{\sigma}_z^{[j]} (\alpha_{\boldsymbol{p}}^{[j]} \hat{B}_{\boldsymbol{p}}^{\dagger} - \text{h.c.})\right] \exp\left[\sum_{\boldsymbol{p}} \Phi_{\boldsymbol{p}}\right], \tag{3a}$$

where

$$\Phi_{\boldsymbol{p}}^{(L)} = \left|\sum_{j=1}^{L} g_{\boldsymbol{p}}^{[j]} \hat{\sigma}_z^{[j]}\right|^2 \frac{\Omega_{\boldsymbol{p}} t - \sin\Omega_{\boldsymbol{p}} t}{\Omega_{\boldsymbol{p}}^2}, \tag{3b}$$

$$\alpha_{\boldsymbol{p}}^{[j]} = g_{\boldsymbol{p}}^{[j]} \frac{1 - \exp({\rm i}\Omega_{\boldsymbol{p}} t)}{\Omega_{\boldsymbol{p}}}. \tag{3c}$$

We introduce the macroscopic polarization operator

$$\tilde{S}_{+} \equiv \sum_{j=1}^{L} \hat{\sigma}_{+}^{[j]} {\rm e}^{{\rm i}\boldsymbol{q}\cdot\boldsymbol{r} + {\rm i}\upsilon_j t}, \tag{4}$$

where $\boldsymbol{q}$ and υ_j describe the wave number vector of the polarization and the energy separation of jth qubit, respectively. The radiation intensity due to this polarization observed in optical spectroscopy is

$$|P^{(L)}(\boldsymbol{q}, t)|^2 = |{\rm Tr}[\tilde{S}_{+} \tilde{\rho}_{\rm QR}^{(L)}(t)] {\rm e}^{-{\rm i}\omega t}|^2 = \frac{1}{4}\exp(-4\Gamma) \times \left|\sum_{Q=1}^{L} f_{Q} \left[\prod_{j=1(\neq Q)}^{L} \cos(4\Theta_{jQ})\right]\right|^2, \tag{5a}$$

where

$$\Gamma = 2\sum_{p} \frac{|g_p|^2}{\Omega_p^2}(1-\cos\Omega_p t)\coth\frac{\hbar\Omega_p}{2k_{\rm B}T}, \tag{5b}$$

$$\Theta_{jQ} = \sum_{p} \Theta_p \cos \boldsymbol{p}\cdot\boldsymbol{r}_{j\rm Q}, \tag{5c}$$

$$\Theta_p = \frac{|g_p|^2}{\Omega_p^2}(\Omega_p t - \sin\Omega_p t), \tag{5d}$$

$$f_Q = \exp\lfloor -{\rm i}({\rm k}-{\rm q})\cdot\boldsymbol{r}_Q + {\rm i}(\upsilon_Q - \omega)t\rfloor, \tag{5e}$$

and $\boldsymbol{r}_{jQ} \equiv \boldsymbol{r}_j - \boldsymbol{r}_Q$ and $g_p^{[j]} \equiv g_p {\rm e}^{-{\rm i}\boldsymbol{p}\cdot\boldsymbol{r}_j}$.

Now let us consider a situation where all qubits exist within a range much shorter compared with the coherence length, taking an approximation $\Theta_{jQ} \to \Theta \equiv \sum_p \Theta_p$ as $\boldsymbol{r}_{jQ} \cong 0$ in Eq. (5c), and one observes the polarization radiation in the direction at $\boldsymbol{k} = \boldsymbol{q}$. The radiation intensity is then given by

$$|P^{(L)}(t)|^2 = \left(\frac{L}{2}\right)^2 \exp(-4\Gamma)\cos^{2(L-1)}4\Theta. \tag{6}$$

Here the Ohmic spectrum

$$I(\Omega) = \sum_{p} \delta(\Omega - \Omega_p)|g_p|^2 = a_0\Omega\exp\left(-\frac{\Omega}{\Omega_{\rm c}}\right), \tag{7}$$

is assumed for the reservoir, where a_0 is the dimensionless coupling constant. $\Omega_{\rm c}$ is the cut-off frequency. Fig. 1 shows the macroscopic polarization normalized at $t = 0$ in the case of L qubits. As seen, increasing the number of qubits, the polarization oscillates and its peak width becomes narrower keeping the same period. Thus the decay of the macroscopic polarization is accelerated as L gets larger. This means that the other qubits are to be regarded as an additional environment coupled with each other via a common reservoir. This beat signal is governed by the factor $\cos^{2(L-1)}4\Theta$, which does not depend on the temperature, but does the spectral density of the reservoir boson. The beat frequency in the Ohmic-model is approximately determined by a factor $f_{\rm b} \cong 4a_{\rm o}\Omega_{\rm c}$.

The accelerated decoherence seen in Fig. 1 is the manifestation of the entanglement between the qubits. Now we evaluate this entanglement by using the Peres criterion [7] in the case of two qubits. Actually the necessary and sufficient criterion for

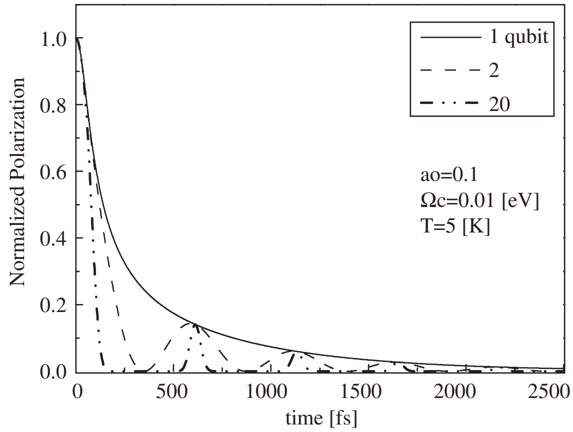

Fig. 1. The decay curve of the macroscopic polarization normalized at $t = 0$ in the case of L qubits and the Ohmic reservoir model. The solid, broken, and dash–dot–dot lines are the calculated results for a single, 2, and 20 qubits, respectively. Parameters are $a_{\rm o} = 0.1$, $\Omega_{\rm c} = 10\,{\rm meV}$, $T = 5\,{\rm K}$. The beat frequency is $f_{\rm b}{\sim}4\,{\rm meV}$.

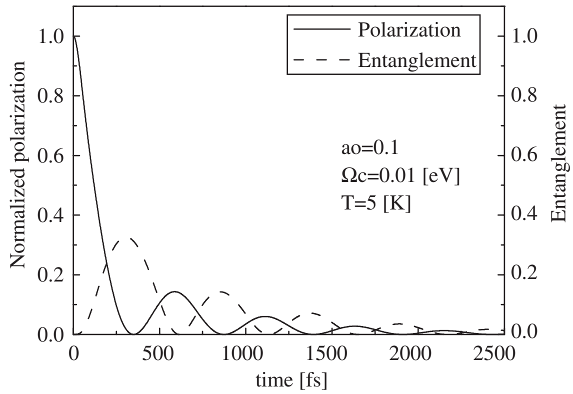

Fig. 2. The time evolution of the normalized polarization (solid line) and the entanglement (broken line) of two qubits. The reservoir model and its parameters are the same as Fig. 1.

the entanglement is known only for qubit–qubit and qubit–qutrit systems [8]. In the case of two-qubit, the entanglement measure $\varepsilon(\hat{\rho})$ is defined as

$$\varepsilon(\hat{\rho}) \equiv -2\sum_{i} \lambda_i^-, \tag{8}$$

based on the negativity of the partial transposition of states, where λ_i^- describes a negative eigenvalue of the partial transposed matrix $\hat{\rho}^{\rm PT}$. The qubits are separable states for $\varepsilon(\hat{\rho}) = 0$, and maximally entangled states for $\varepsilon(\hat{\rho}) = 1$. Fig. 2 shows the time evolution of the entanglement (dashed line) and polarization decoherence (solid line) of two qubits in the case of $\boldsymbol{r}_{jQ}\sim 0$. As seen, the entanglement measure varies almost in antiphase with the beat signal of the polarization, and relaxes to separable states eventually. The entanglement thus induced via the reservoir does not attain the maximal $\varepsilon(\hat{\rho}) = 1$, in principle. In fact, perfect entanglement of the qubits is possible only when they are completely isolated from the environment. (This is actually the principle used for the security proof of quantum cryptography.) In contrast, the reservoir-induced entanglement of qubits cannot exist apart from the coupling to the reservoir. So it is not straightforward to utilize such an entanglement for quantum information processing. In Ref. [4], a possibility is presented which distills perfect entanglement from many imperfect entangled pairs created via the reservoir. This will hardly be practical at present. Rather we will apply the analysis of the reservoir-induced entanglement of qubits to precise estimation of the reservoir characteristics. But before that, we should interpret the mechanism of the beat signal.

The direct origin of the beat signal is the so-called Lamb phase shift, i.e. the factor given by Eq. (3b). This factor cancels out in the qubit density matrix in the case where the qubits couple independently to separate reservoirs ($L = 1$ in Fig. 1). In the present case, on the other hand, it causes a significant effect on the dynamics, inducing the phase shift depending on the quantum state of qubit array. To see this effect more explicitly, we set $\Gamma = 1$ by hand, and extract pure state component. The state evolution is then written as $|01\rangle + |10\rangle + \exp(4i\Theta(|00\rangle + |11\rangle))$. Thus the reservoir induces the relative phase shift between the anti-parallel and parallel state components of qubits.

The beat signal thus caused has some unique nature. Firstly the decay curve of the collective polarization as shown in Fig. 1 has the peak structure whose peak positions are determined by the factor $f_{\mathrm{b}} \cong 4a_{\mathrm{o}}\Omega_{\mathrm{c}}$ and the peak width narrows rapidly as the number of qubits interacting with the common reservoir bosons increases. This implies that the peaks will be sharpened as the temperature is lowered, since the coherence length of the reservoir will extend at lower temperature and the number of qubits within the coherence length will increase. Secondly such a behavior does not depend on the optical polarization of exciting pulses, because it is due to the mutual interaction between the identical qubits, not like the interference between the degenerate levels of relevant excitations behind a qubit. Based on these features one will be able to distinguish the effect of our interest from other kinds of beat signals encountered frequently in solid-state systems. Once this effect is observed, one can evaluate $a_{\mathrm{o}}\Omega_{\mathrm{c}}$ from the peak distance. The cutoff frequency Ω_{c} itself can be estimated by the time scale in which the polarization decays completely at a low-temperature regime. So the coupling constant a_{o} can be estimated. Thus one can identify the reservoir characteristics.

Finally we study the polarization decay in the two-qubit case, considering the finite distance between the qubits. Introducing the transit time 't_{s}' as a measure of the qubit distance as $\boldsymbol{p}\cdot\boldsymbol{r}_{12} \equiv \Omega_{\boldsymbol{p}} t_{\mathrm{s}}$, Eq. (5a) can be rewritten as

$$|P^{(2)}(t, t_{\mathrm{s}})|^2 = \cos^2 4\Theta_{\mathrm{s}} \exp(-4\Gamma), \tag{9a}$$

$$\Theta_{\mathrm{s}} \equiv \sum_{\boldsymbol{p}} \Theta_{\boldsymbol{p}} \cos \Omega_{\boldsymbol{p}} t_{\mathrm{s}}. \tag{9b}$$

Fig. 3 shows the polarization decay curves for tree kinds of the transit times. As the transit time (the

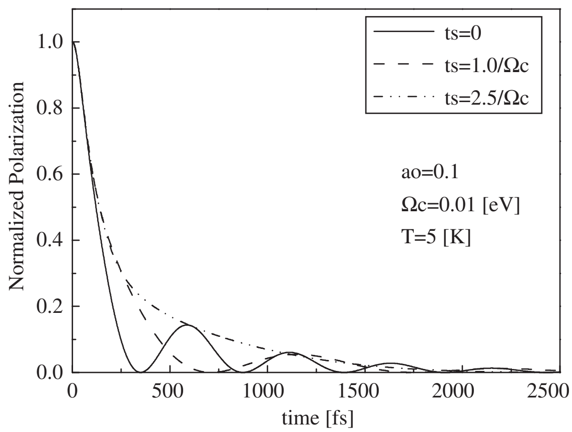

Fig. 3. The time evolution of the normalized polarization of two qubits in the case of $n = 0$ (solid line), $n = 1.0$ (broken line), and $n = 2.5$ (dash–dot–dot line) in setting $t_{\mathrm{s}} = n \times \Omega_{\mathrm{c}}$.

qubit distance) increases, the period of oscillation gets longer, and disappears when the qubits are apart over the coherence length of the reservoir, as expected. This behavior is also to be tested using several samples with different qubit densities.

3. Conclusions

We have analyzed the decoherence of multi-qubit system interacting with a common Ohmic reservoir. The qubits are entangled by exchanging the reservoir bosons within the memory time. This interaction induces the Lamb phase shift on the qubit array. As a result, the decay curve of the macroscopic polarization shows the beat signal. We have quantified the reservoir-induced entanglement and show how this can be evaluated by measuring the polarization decay. This phenomenon has never been observed in laboratory. So it must be put to experimental test. We have discussed how our analysis can be used to estimate the reservoir characteristics. To know the reservoir characteristics is actually the very first step to design methods of decoherence stabilization for quantum information processing [6].

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